# DoE for Scale-Up/Down

## *… using MODDE®, the DoE-DiVa®*
## *and a little bit of R*

**Session 4: Scale Down of a Bioreactor -- 22.02. 2023**

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umesoft

Gefördert durch:

Bundesministerium für Wirtschaft und Klimaschutz

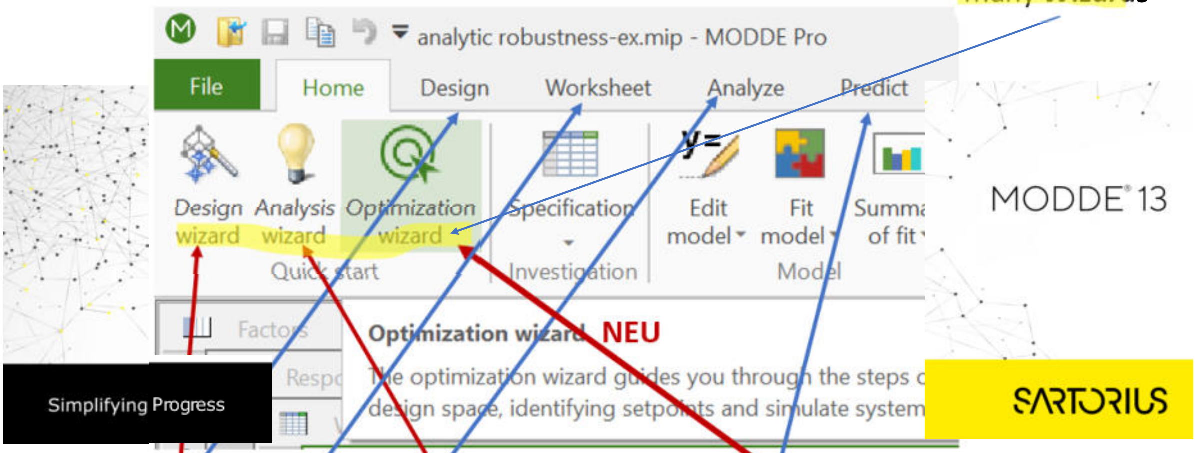
aufgrund eines Beschlusses des Deutschen Bundestages

## The roles of ***MODDE®*** , ***DoE-DiVa®*** and ***R***

- ***MODDE®*** for the ***complete DoE-workflow*** of Design, Analysis, Prediction and Optimization, QbD, etc.
- ***DoE-DiVa*** introduces $u \xrightarrow{T_x} x$
  *U̲ser-factors **and** eX̲plaining-factors and designs for these*
- ***R*** is used for prototyping and for validating of ***DoE-DiVa*** calculations, and for developping *DoE-DiVa Help System*.

# This is how ***MODDE®*** looks.

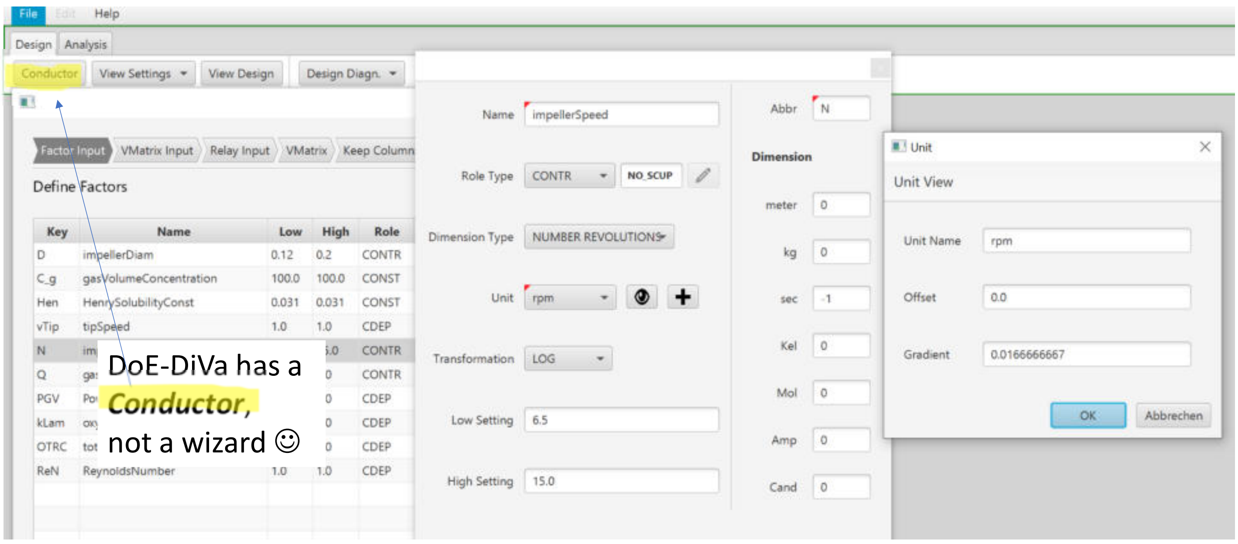
MODDE® has many ***Wizards***




**1.Design** **2. Work** **3. Analyze** **4. Predict&optimize**

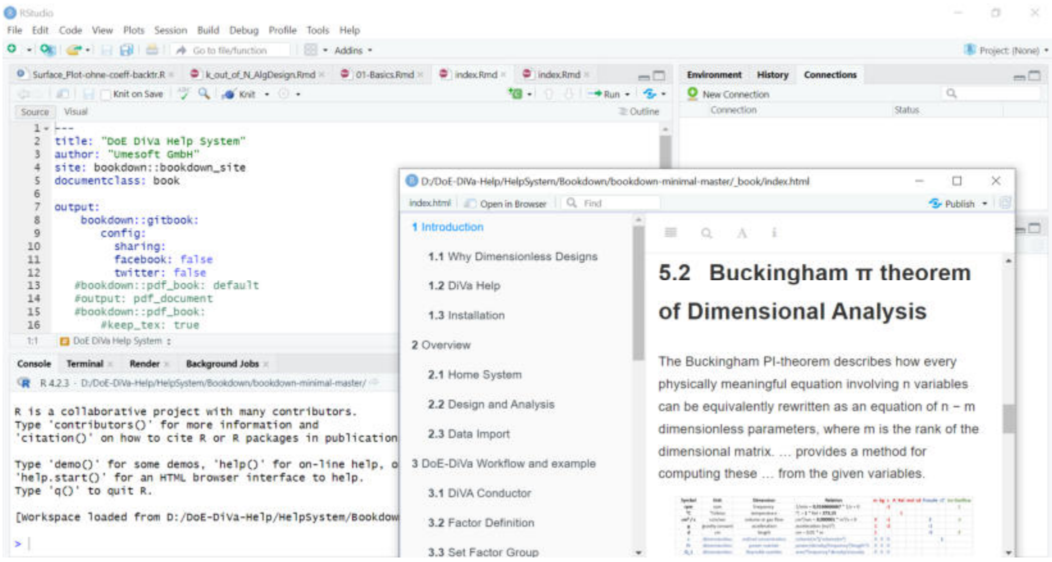


# This is how the ***DoE-DiVa*** looks.



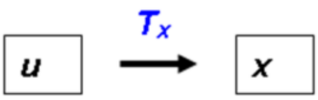

DoE-DiVa has a ***Conductor***, not a wizard ☺

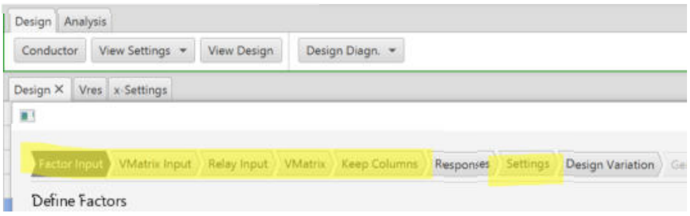
# This is how the ***R**(-Studio)* looks.






# ***DoE-DiVa's*** focus is on factors



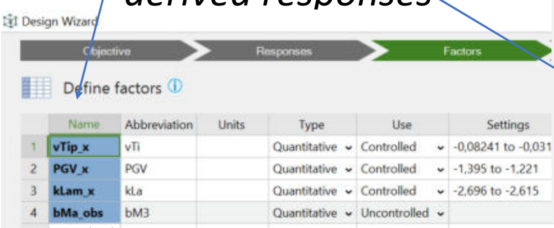



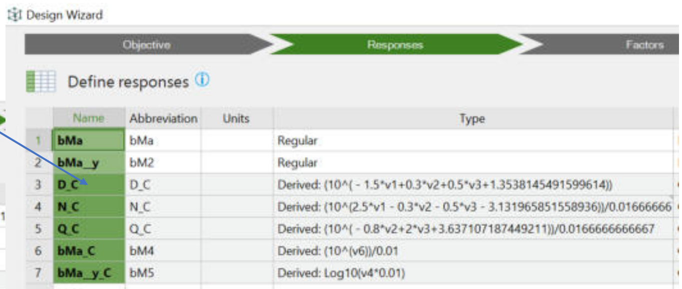

In ***MODDE®***

***X**-factors* act as *factors*

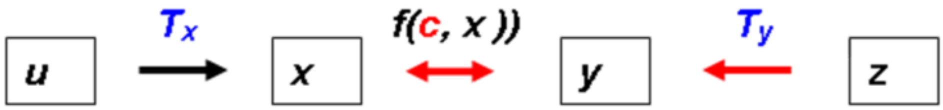
***U**ser-factors* become

*derived responses*



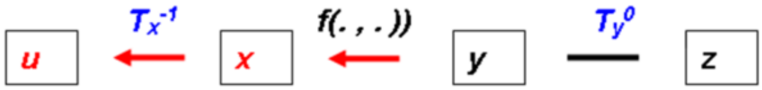
# The ***DoE-DiVa***-approach



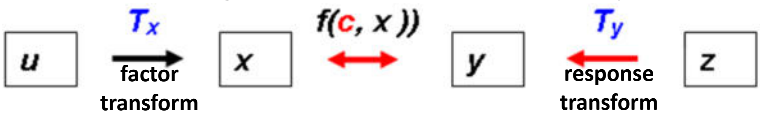

***u: U****ser-factor,* to be set in the experiment, e.g. *Temp, pressure etc.*
***x:*** *e****X****plaining-factor,* to be used in the model, e.g. *a force-ratio*
$T_x$*:* *transformation* to get from ***u*** to ***x***, e.g. *ratio, dimensionless variable*
***c:*** *coefficients or parameters in the model,* ***f****, to be determined by model FIT*
***z:*** *measured response variable*
***y:*** *transformed response value, e.g. ratio or product of a* ***z*** *and some* ***u***
$T_y$*:* *transformation* to get from ***z*** to ***y***, *may also just be log or neg-log*

# The role of the *Transformations*

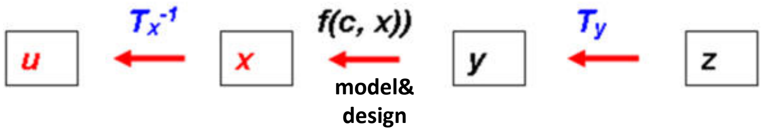
**Design: How to get *u* from *f* and the *tr*ansformation**




**Model Fit: how to get coefficients, *c*, from *x* and *y***




**Optimization: how to get the *u* from the specification for *z***

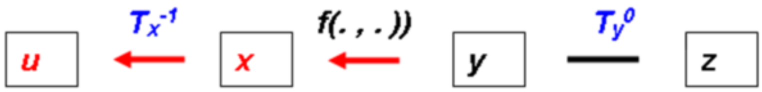





# The *Approximation* Trick to get $T_x^{-1}$

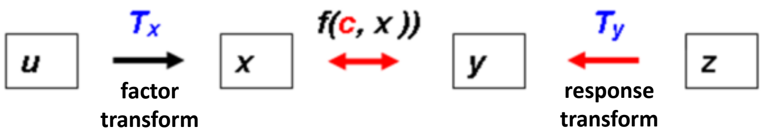
Session 3

**Design: To get u from *x*, $T_x$ has to be inverted**




**Inverting a non-linear $T_x$ may be difficult or impossible, so $T_x$ is approximated by a linear $T_{approx}$ and *when we write $T_x^{-1}$ we mean $T_{approx}^{-1}$***

**Model Fit: To get *x* from *u* for Fit, $T_x$ need not be inverted, $T_{approx}$ is not needed.**

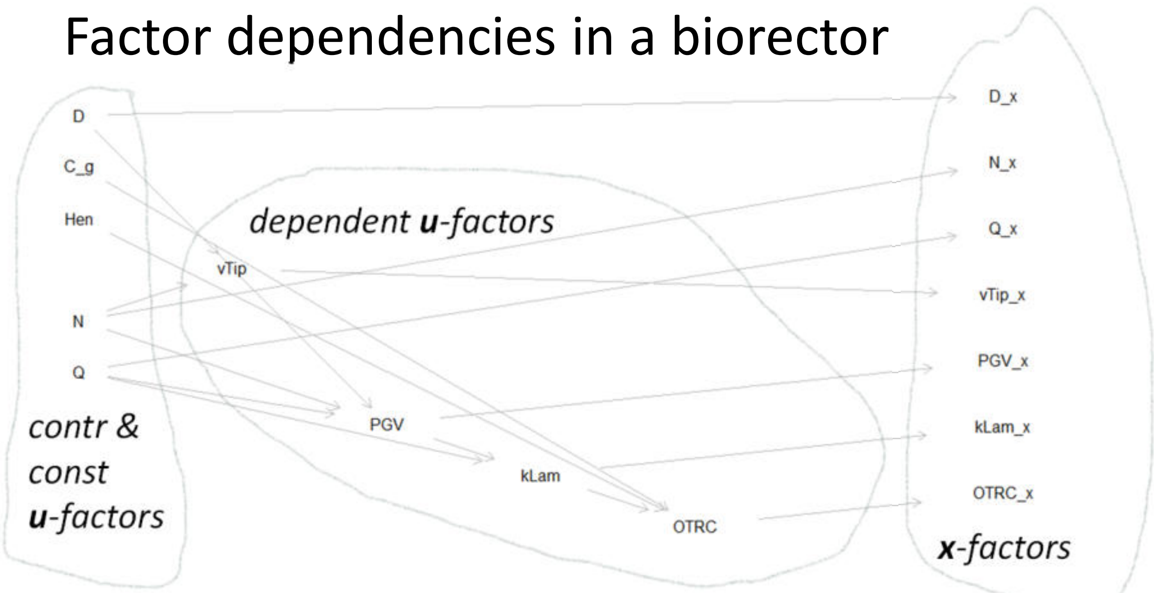


# Different types of *factor dependencies*

- *Quotients and Products* of controlled (or constant) ***u****-factors* can be directly used as ***x****-factors,* **HOWEVER** it may be advisable – in the following situations – to first introduce ***dependent u-factors:***
- ***Nested formulae***: *Oxygen transfer capacity*, ***OTRC***, depends on *oxygen transfer coefficient*, ***kLa***, which again depends on *power to volume ratio*, ***PGV***, and *gas flow rate*, ***Q***.
- ***Non-linear formulae:*** *Oxygen transfer capacity*, ***OTRC,*** is the sum of products of ***kLa****'s* and *concentration gradients*. These will be ***approximated*** when determining $T_{approx}$ and $T_{approx}^{-1}$
- ***Data tables*** from prior *experimental* or *simulation* runs are *available*, but not *as forumulae*, typically ***simulation results for the high scale*** and ***experimental results from the low scale***.



# Factor dependencies in a biorector



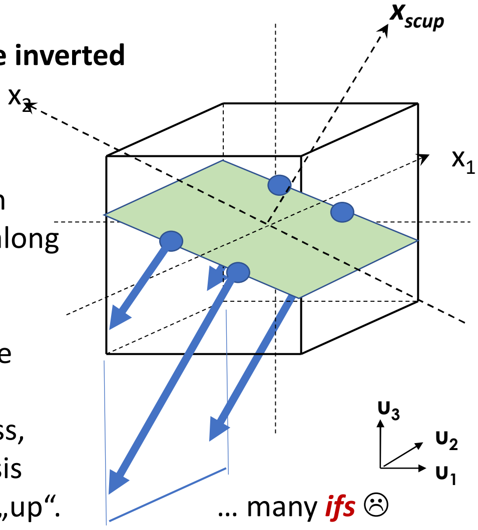
# „Simple“ ***Scale Up*** using Similarity

Session 2

**Design: To get u from x, $T_x$ has to be inverted**

*u* ← $T_x^{-1}$ ← *x*

***If*** there are ***fewer x**-factors* than ***u**-factors*, then there is an orthogonal $x_{scup}$-“slider“-direction, along which both ***x***-coordinates stay the same.
$T_x^{-1}$ (respectively $T_{approx}^{-1}$) then depends on the „sliding-direction“, „down“ or „up“. ***If*** the ***x****‘s* are ***dimensionless***, and ***if*** the ***u*** give a ***„complete“ description*** of all relevant aspects of the process, the Similarity Principle of Dimensionless Analysis permits the transfer of results from „down“ to „up“.




… many ***ifs*** ☹



# „Complex“ ***Scale Up*** using „All that one knows“
*„A-t-o-k approach“*

TODAY:

**Design: To get u from x, $T_x$ has to be inverted**

*u* ← $T_x^{-1}$ *at high Scale* ← *x*

Challenge: They may be different

The back transform, $T_x^{-1}$ (respectively $T_{approx}^{-1}$), may be ***much more different*** at *Low Scale* than at *High Scale* ***than just a different „slider position“***, particularly if there are
- ***differing low/high intervals*** for (geometrical) ***u****-factors,*
- ***different u-factor*** variables altogether (=> a different dimension(!)),
- ***different formulae*** or *data* relating relevant ***u****- and* ***x****-factor* variables.

*u* ← $T_x^{-1}$ *at low Scale* ← *x*

# ***u-factors*** (at **high** scale)

| | Name | Abbr | group | Role | userunit | transform | userlow | userhigh |
|---|---|---|---|---|---|---|---|---|
| 1 | impellerDiameter | D | D | contr | m | log | 0,4 | 0,8 |
| 2 | gasVolumeConcentration | C_g | C_g | const | % | log | 21 | 21 |
| 3 | HenrySolubilityConstant (conc/conc) | Hen | Hen | const | [-] | log | 0,031 | 0,031 |
| 4 | tip speed | vTip | vTip | cdep | m/s | log | 1 | 1 |
| 5 | impellerSpeed | N | N | contr | 1/s | log | 1,3 | 2,6 |
| 6 | gasFlowRate | Q | Q | contr | m^3/min | log | 10 | 20 |
| 7 | PowerGVolumeRatio | PGV | PGV | cdep | W/m^3 | log | 1 | 1 |
| 8 | oxygenTransferCoeffiMixedZone | kLam | kLam | cdep | 1/s | log | 0,0006 | 0,006 |
| 9 | oxygenTransferCoeffiBubbleZone | kLab | kLab | cdep | 1/s | log | 1 | 1 |
| 10 | total oxygen transfer capacity | OTRC | OTRC | cdep | 1/s | log | 1 | 1 |
| 11 | ReynoldsNumber | ReN | ReN | cdep | [-] | log | 1 | 1 |

At high scale, there are ***two zones***, regions or „compartments“, ***mixed zone*** and ***bubble-zone:*** *oxygen transfer coefficient, kla, is* ***different in the two zones***

Values at **high scale** for ***user low*** and ***high*** for *geometric factors* are typically ***different*** at **low scale**, whereas *intinsic properties* are ***scale independent***

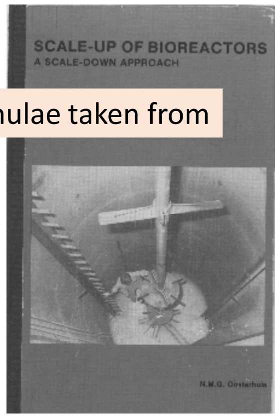
# *u-formulae* (at **high** scale)

| | formula |
|---|---|
| 1 | vTip=D*N |
| 2 | PGV=D^5*N^3*Q^(-0.5) |
| 3 | **kLam**=0.0323*(PGV)^0.4*((Q*0.0166666667)/3.14159*4/2.4^2)^0.5 |
| 5 | **kLab**=0.32*((Q*0.0166666667)/3.14159*4/2.4^2)^0.7 |
| 4 | **OTRC**=kLam*(C_g*0.01)*Hen+kLab*(C_g*0.01)*Hen |
| 6 | ReN=N*D^2/1.470588e-06 |

The formulae for *oxygen transfer coefficients, kla**m/b***, are ***different in the two zones, mixed*** *zone* and ***b****ubble zone*.

The formula for total *oxygen transfer capacity*, *OTRC*, ***combines results of the two zones***.

Formulae taken from

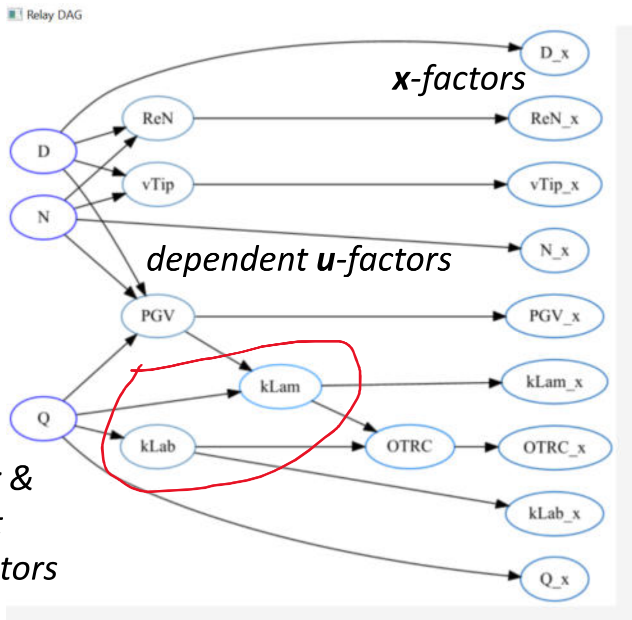



**Proefschrift** ter verkrijging van de graad van doctor in de technische wetenschappen aan de Technische Hogeschool Delft, op gezag van de Rector Magnificus, prof.ir. B.P.Th. Veltman, in het openbaar te verdedigen ten overstaan van het College van Dekanen op 20 maart 1984 te 16.00 uur door **Nicolaas Marius Gerard Oosterhuis,** geboren te Zaandijk. Landbouwkundig ingenieur.



# Nested ***u-factor*** *dependencies*

(at **high** scale)



# *u-factors*

(at low scale)

| | Name | Abbr | group | Role | userunit | transform | userlow | userhigh |
|---|---|---|---|---|---|---|---|---|
| 1 | impellerDiameter | D | D | contr | m | log | 0,08 | 0,16 |
| 2 | gasVolumeConcentration | C_g | C_g | const | % | log | 21 | 21 |
| 3 | HenrySolubilityConstant (conc/conc) | Hen | Hen | const | [-] | log | 0,031 | 0,031 |
| 4 | tip speed | vTip | vTip | cdep | m/s | log | 1 | 1 |
| 5 | impellerSpeed | N | N | contr | 1/s | log | 6,5 | 15 |
| 6 | gasFlowRate | Q | Q | contr | m^3/min | log | 1 | 2 |
| 7 | PowerGVolumeRatio | PGV | PGV | cdep | W/m^3 | log | 1 | 1 |
| 8 | oxygenTransferCoeffiMixedZone | kLam | kLam | cdep | 1/s | log | 1 | 1 |
| 9 | total oxygen transfer capacity | OTRC | OTRC | cdep | 1/s | log | 1 | 1 |
| 10 | ReynoldsNumber | ReN | ReN | cdep | [-] | log | 2 | 1 |

At low scale, there is only ***one zone***, region or „compartment“, the ***mixed zone*** and there is ***no bubble-zone:***
*oxygen transfer coefficient, klam,* ***only exists once***, there is no *klab.*

Values At **low scale** for ***user low*** and ***high*** for *geometric* factors are typically different at **high scale**, whereas intrinsic *properties* are scale independent



# *u-dependency formulae*

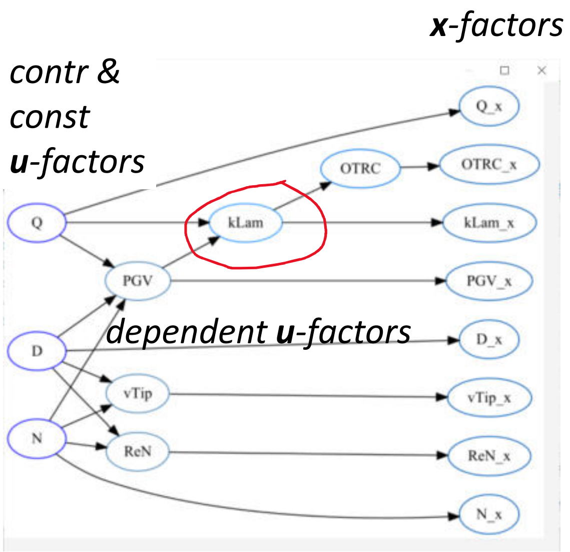
(at low scale)

| | formula |
|---|---|
| 1 | vTip=D*N |
| 2 | PGV=D^5*N^3*Q^(-0.5) |
| 3 | **kLam**=0.0323*(PGV)^0.4*((Q*0.0166666667)/3.14159*4/2.4^2)^0.5 |
| 4 | **OTRC**=kLam*((C_g*0.01)*Hen) |
| 5 | ReN=N*D^2/1.470588e-06 |

There is only one formula for *oxygen transfer coefficient, kla**m**,* because there is only ***one zone,*** the *mixed zone*.
The formula for total *oxygen transfer capacity*, *OTRC*, is simpler.

# Nested *u-factor* dependencies

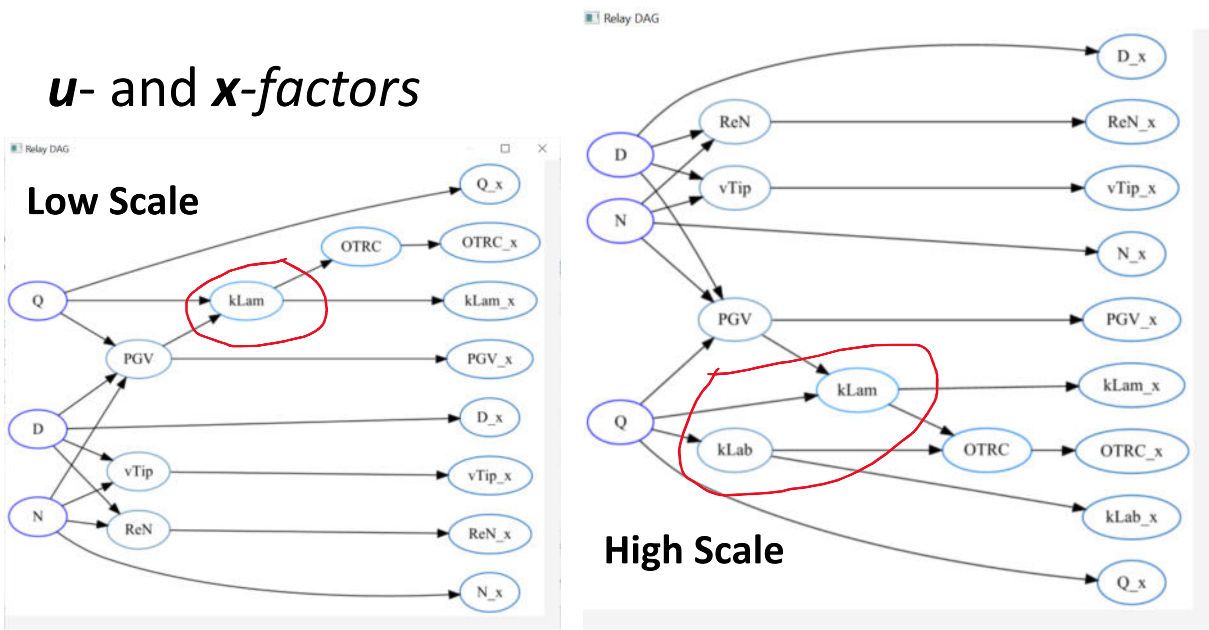
(at **low** scale)






1. The DoE-Diva approach
2. Bioreactor at High and Low Scale
3. ***Preparing Scale Down using DoE-DiVa***
4. Performing Scale Down using MODDE®

# *u*- and *x*-*factors*






# Step 1: *u-factors* *(high scale first)*

Factor Input | VMatrix Input | Relay Input | VMatrix | Keep Columns | Responses | Settings | Design Variation | Generate Design

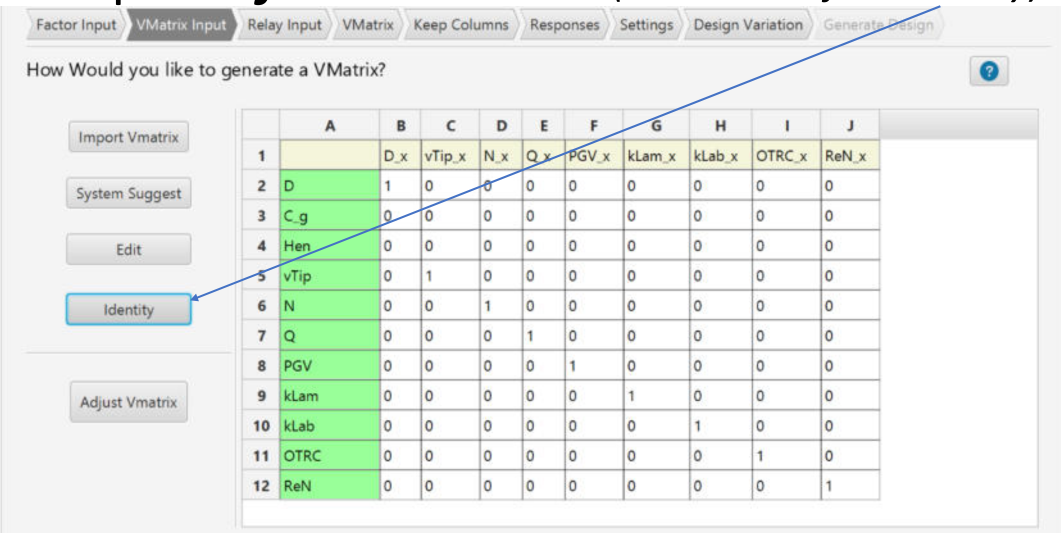
Define Factors

| Key | Name | Low | High | Role | Unit | Transformation | Dimension |
|---|---|---|---|---|---|---|---|
| D | impellerDiam | 0.4 | 0.8 | CONTR | m | LOG | LENGTH |
| C_g | gasVolumeConcentration | 21.0 | 21.0 | CONST | % | LOG | DIMENSION_LESS |
| Hen | HenrySolubilityConst | 0.031 | 0.031 | CONST | SI | LOG | DIMENSION_LESS |
| vTip | tipSpeed | 1.0 | 1.0 | CDEP | m/s | LOG | SPEED |
| N | impellerSpeed | 1.3 | 2.6 | CONTR | revpm | LOG | NUMBER_OF_REVOL... |
| Q | gasFlowRate | 10.0 | 20.0 | CONTR | m^3/min | LOG | VOLUME_FLOW |
| PGV | PowerGVolumeRatio | 1.0 | 1.0 | CDEP | W/m^3 | LOG | POWER DENSITY |
| kLam | oxygenTransferCoeffiMixedZone | 6.0E-4 | 0.006 | CDEP | 1/s | LOG | REACTION_RATE1 |
| kLab | oxygenTransferCoeffiBubbleZone | 1.0 | 1.0 | CDEP | SI | LOG | REACTION_RATE1 |
| OTRC | totalOxygenTransferCapacity | 1.0 | 1.0 | CDEP | 1/s | LOG | REACTION_RATE1 |
| ReN | ReynoldsNumber | 1.0 | 1.0 | CDEP | SI | LOG | DIMENSION_LESS |

There are 3 *controlled* ***u****-factors*

There are 6 *dependent* ***u****-factors*

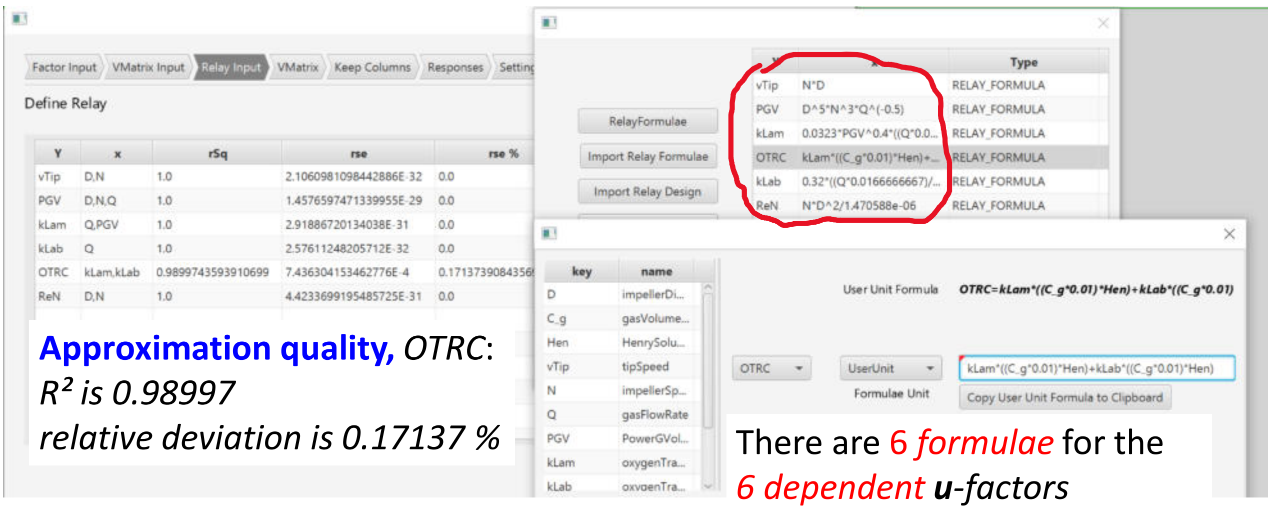
# Step 2: ***x-factors*** *(in this case just identity)*




| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | D_x | vTip_x | N_x | Q_x | PGV_x | kLam_x | kLab_x | OTRC_x | ReN_x |
| 2 | D | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | C_g | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Hen | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | vTip | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | N | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | Q | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 8 | PGV | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 9 | kLam | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 10 | kLab | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 11 | OTRC | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 12 | ReN | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |



# Step 3: ***u-factor dependencies*** *(OTRC at **high** scale)*




| Y | x | rSq | rse | rse % |
|---|---|---|---|---|
| vTip | D,N | 1.0 | 2.1060981098442886E-32 | 0.0 |
| PGV | D,N,Q | 1.0 | 1.4576597471339955E-29 | 0.0 |
| kLam | Q,PGV | 1.0 | 2.91886720134038E-31 | 0.0 |
| kLab | Q | 1.0 | 2.57611248205712E-32 | 0.0 |
| OTRC | kLam,kLab | 0.9899743593910699 | 7.436304153462776E-4 | 0.171373908435 |
| ReN | D,N | 1.0 | 4.4233699195485725E-31 | 0.0 |

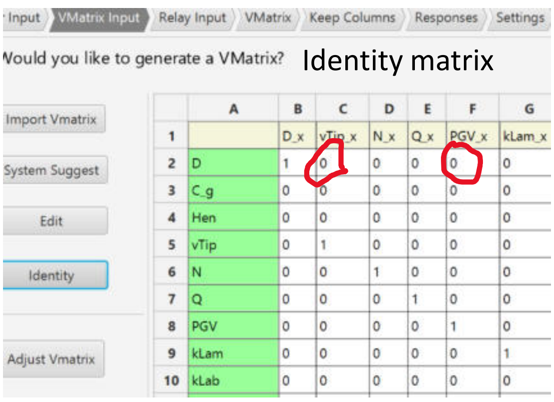
| Y | x | Type |
|---|---|---|
| vTip | N*D | RELAY_FORMULA |
| PGV | D^5*N^3*Q^(-0.5) | RELAY_FORMULA |
| kLam | 0.0323*PGV^0.4*((Q*0.0... | RELAY_FORMULA |
| OTRC | kLam*((C_g*0.01)*Hen)+... | RELAY_FORMULA |
| kLab | 0.32*((Q*0.0166666667)/... | RELAY_FORMULA |
| ReN | N*D^2/1.470588e-06 | RELAY_FORMULA |

| key | name |
|---|---|
| D | impellerDi... |
| C_g | gasVolume... |
| Hen | HenrySolu... |
| vTip | tipSpeed |
| N | impellerSp... |
| Q | gasFlowRate |
| PGV | PowerGVol... |
| kLam | oxygenTra... |
| kLab | oxygenTra... |

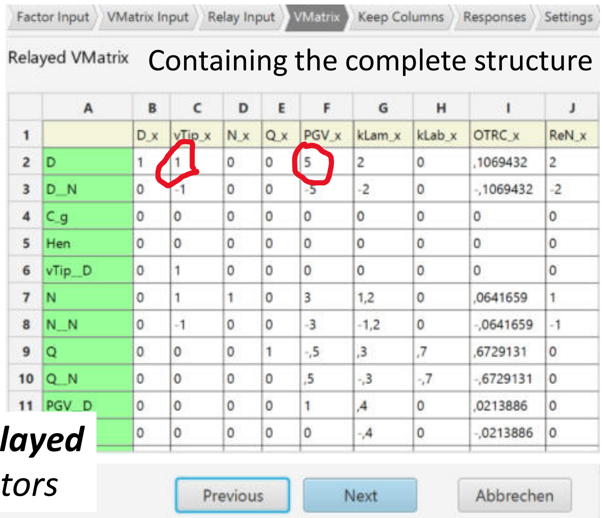
**Approximation quality,** *OTRC*:
*$R^2$ is 0.98997*
*relative deviation is 0.17137 %*

There are *6 formulae* for the *6 dependent **u**-factors*

# 4: Info, $T_{approx}$ – *transformation as a matrix*




| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | D_x | vTip_x | N_x | Q_x | PGV_x | kLam_x |
| 2 | D | 1 | 0 | 0 | 0 | 0 | 0 |
| 3 | C_g | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Hen | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | vTip | 0 | 1 | 0 | 0 | 0 | 0 |
| 6 | N | 0 | 0 | 1 | 0 | 0 | 0 |
| 7 | Q | 0 | 0 | 0 | 1 | 0 | 0 |
| 8 | PGV | 0 | 0 | 0 | 0 | 1 | 0 |
| 9 | kLam | 0 | 0 | 0 | 0 | 0 | 1 |
| 10 | kLab | 0 | 0 | 0 | 0 | 0 | 0 |

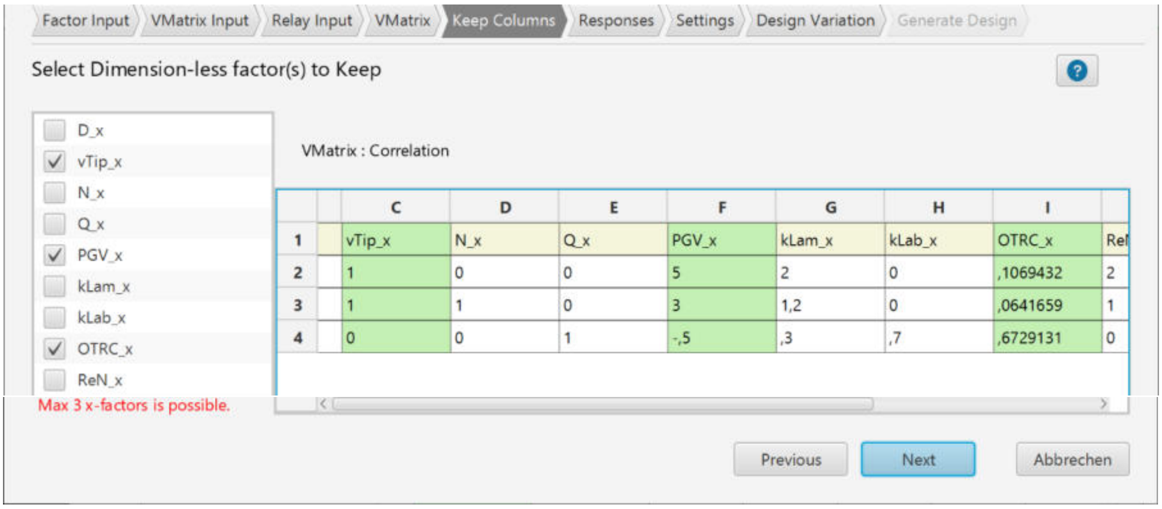



| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | D_x | vTip_x | N_x | Q_x | PGV_x | kLam_x | kLab_x | OTRC_x | ReN_x |
| 2 | D | 1 | 1 | 0 | 0 | 5 | 2 | 0 | ,1069432 | 2 |
| 3 | D_N | 0 | -1 | 0 | 0 | -5 | -2 | 0 | -,1069432 | -2 |
| 4 | C_g | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | Hen | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | vTip_D | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | N | 0 | 1 | 1 | 0 | 3 | 1,2 | 0 | ,0641659 | 1 |
| 8 | N_N | 0 | -1 | 0 | 0 | -3 | -1,2 | 0 | -,0641659 | -1 |
| 9 | Q | 0 | 0 | 0 | 1 | -,5 | ,3 | ,7 | ,6729131 | 0 |
| 10 | Q_N | 0 | 0 | 0 | 0 | ,5 | -,3 | -,7 | -,6729131 | 0 |
| 11 | PGV_D | 0 | 0 | 0 | 0 | 1 | ,4 | 0 | ,0213886 | 0 |
| | | 0 | 0 | 0 | 0 | 0 | -,4 | 0 | -,0213886 | 0 |

***u**-factor **dependencies*** have been ***relayed***
from the *contr **u**-factors* to the ***x**-factors*

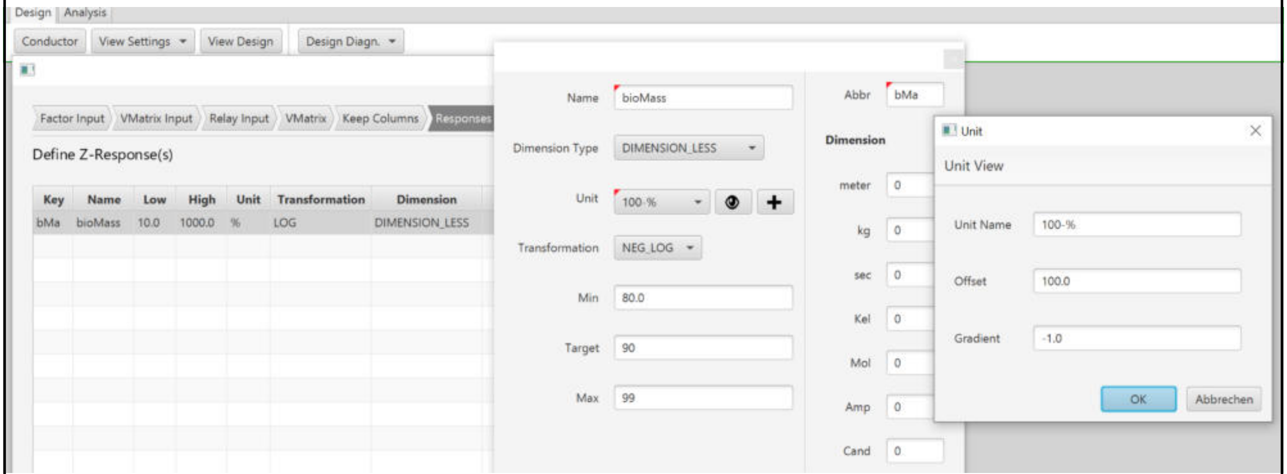


# Step 5: *choose x-factors to use*



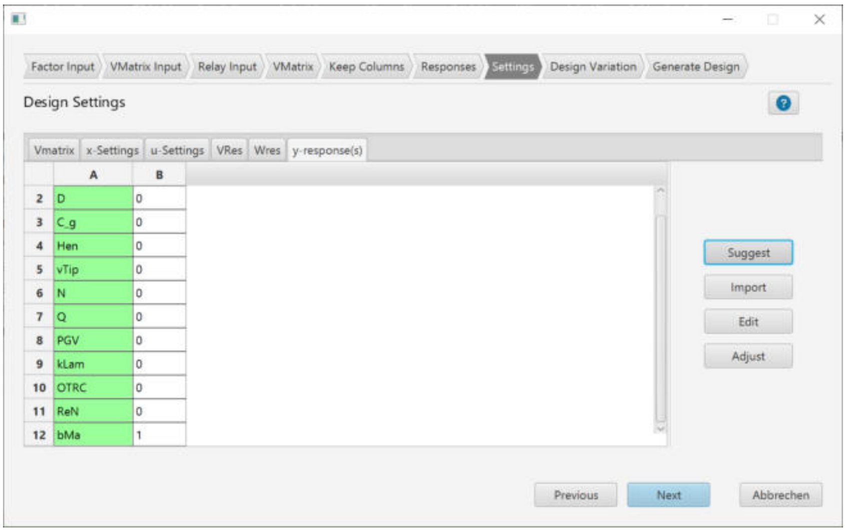

| | C | D | E | F | G | H | I | |
|---|---|---|---|---|---|---|---|---|
| 1 | vTip_x | N_x | Q_x | PGV_x | kLam_x | kLab_x | OTRC_x | Re |
| 2 | 1 | 0 | 0 | 5 | 2 | 0 | ,1069432 | 2 |
| 3 | 1 | 1 | 0 | 3 | 1,2 | 0 | ,0641659 | 1 |
| 4 | 0 | 0 | 1 | -,5 | ,3 | ,7 | ,6729131 | 0 |

# Step 6: ***define z-response(s)*** *(of no relevance today)*






# Step 7: Settings: ***define y-response(s)*** *(for completeness)*



## Step 7f: Settings: *view* $T_{approx}$ *and its inverse* $T_{approx}^{-1}$

atrix | x-Settings | u-Settings | VRes | Wres | y-response(s)

| A | B | C | D | E | F |
|---|---|---|---|---|---|
| | vTip_x | PGV_x | OTRC_x | D_N | C_g |
| D | 1 | 5 | ,1069432 | 0 | 0 |
| D_N | -1 | -5 | -,1069432 | 1 | 0 |
| C_g | 0 | 0 | 0 | 0 | 1 |
| Hen | 0 | 0 | 0 | 0 | 0 |
| vTip_D | 1 | 0 | 0 | 0 | 0 |
| N | 1 | 3 | ,0641659 | 0 | 0 |
| N_N | -1 | -3 | -,0641659 | 0 | 0 |
| Q | 0 | -,5 | ,6729131 | 0 | 0 |
| Q_N | 0 | ,5 | -,6729131 | 0 | 0 |

atrix | x-Settings | u-Settings | VRes | Wres | y-response(s)

| A | B | C | D | E | F | G | H | I |
|---|---|---|---|---|---|---|---|---|
| | D | D_N | C_g | Hen | vTip_D | N | N_N | Q |
| vTip_x | -1,5 | 0 | 0 | 0 | 0 | 2,5 | 0 | ,0000001 |
| PGV_x | ,492178 | 0 | 0 | 0 | 0 | -,492178 | 0 | -,0312879 |
| OTRC_x | ,365707 | 0 | 0 | 0 | 0 | -,365707 | 0 | 1,4628279 |
| D_N | 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| C_g | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| Hen | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| vTip_D | 1,5 | 0 | 0 | 0 | 1 | -2,5 | 0 | -,0000001 |
| N_N | 0 | 0 | 0 | 0 | 0 | 1 | 1 | 0 |
| Q_N | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

These tables can be used later, if it becomes necessary to adapt *low/high settings* of ***u**-factors* at low scale.



## Step 7ff: Settings: *view and edit x-settings to use*

Vmatrix | x-Settings | u-Settings | VRes | Wres | y-response(s)

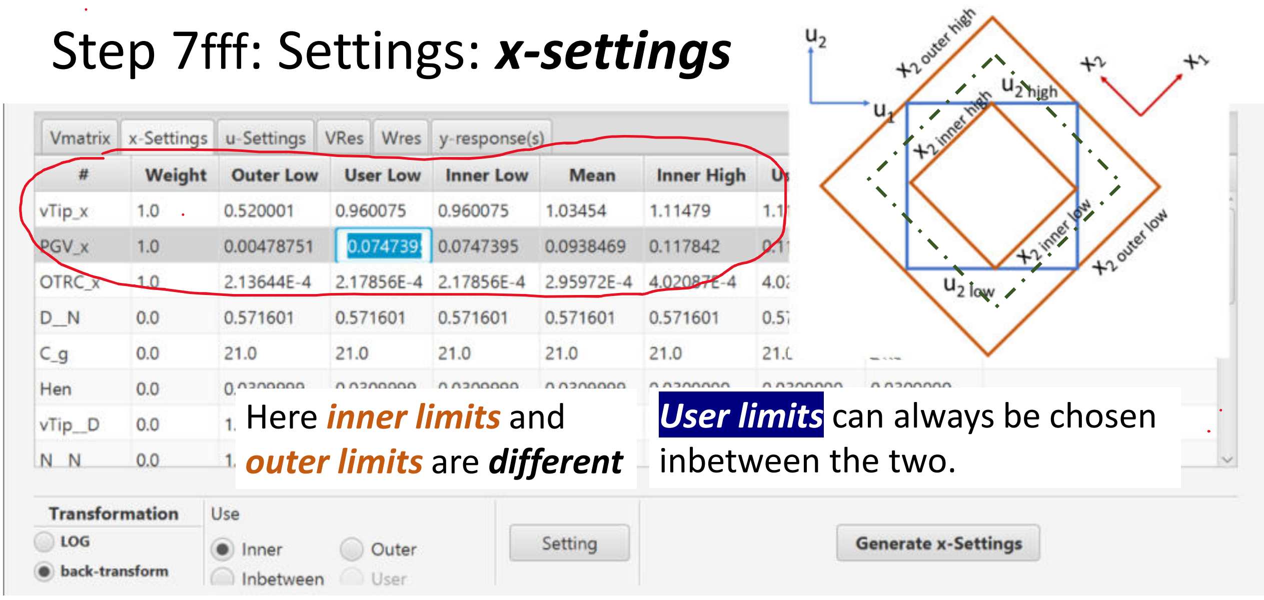
| # | Weight | Outer Low | User Low | Inner Low | Mean | Inner High | User High | Outer High |
|---|---|---|---|---|---|---|---|---|
| vTip_x | 1.0 | 0.520001 | 0.960075 | 0.960075 | 1.03454 | 1.11479 | 1.11479 | 2.08 |
| PGV_x | 1.0 | 0.00478751 | 0.074739 | 0.0747395 | 0.0938469 | 0.117842 | 0.117842 | 1.73327 |
| OTRC_x | 1.0 | 2.13644E-4 | 2.17856E-4 | 2.17856E-4 | 2.95972E-4 | 4.02087E-4 | 4.02087E-4 | 3.83504E-4 |
| D_N | 0.0 | 0.571601 | 0.571601 | 0.571601 | 0.571601 | 0.571601 | 0.571601 | 0.571601 |
| C_g | 0.0 | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 |
| Hen | 0.0 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 |
| vTip_D | 0.0 | 1.03454 | 1.03454 | 1.03454 | 1.03454 | 1.03454 | 1.03454 | 1.03454 |
| N_N | 0.0 | 1.80989 | 1.80989 | 1.80989 | 1.80989 | 1.80989 | 1.80989 | 1.80989 |

**Transformation**: ○ LOG ● back-transform

Use: ● Inner ○ Outer ○ Inbetween ○ User

[Setting] [Generate x-Settings]

# Step 7fff: Settings: ***x-settings***

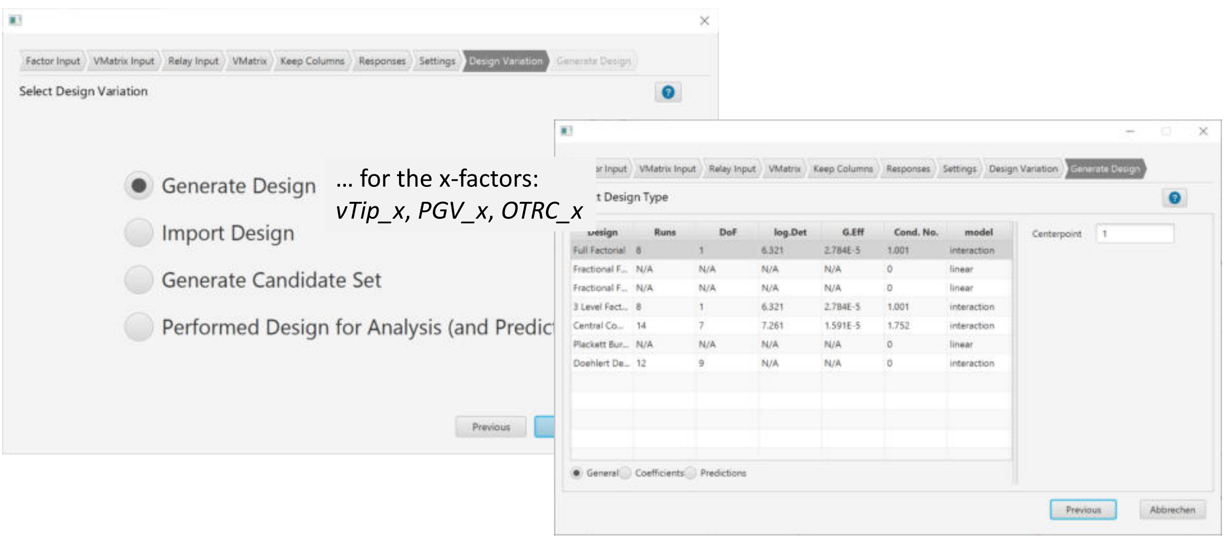



Here ***inner limits*** and ***outer limits*** are ***different***

***User limits*** can always be chosen inbetween the two.

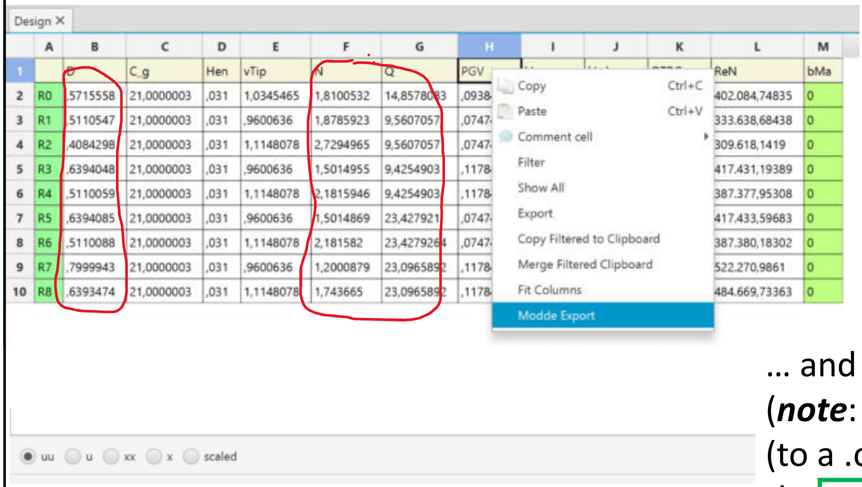


# Step 8: ***Select the design variation***




... for the x-factors: *vTip_x, PGV_x, OTRC_x*

# Step 8f: *Look at the design (uu-design)*




| | A | B | C | D | E | F | G | H | L | M |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | D | C_g | Hen | vTip | N | Q | PGV | ReN | bMa |
| 2 | R0 | ,5715558 | 21,0000003 | ,031 | 1,0345465 | 1,8100532 | 14,8578083 | ,0938 | 402.084,74835 | 0 |
| 3 | R1 | ,5110547 | 21,0000003 | ,031 | ,9600636 | 1,8785923 | 9,5607057 | ,0747 | 333.638,68438 | 0 |
| 4 | R2 | ,4084298 | 21,0000003 | ,031 | 1,1148078 | 2,7294965 | 9,5607057 | ,0747 | 309.618,1419 | 0 |
| 5 | R3 | ,6394048 | 21,0000003 | ,031 | ,9600636 | 1,5014955 | 9,4254903 | ,1178 | 417.431,19389 | 0 |
| 6 | R4 | ,5110059 | 21,0000003 | ,031 | 1,1148078 | 2,1815946 | 9,4254903 | ,1178 | 387.377,95308 | 0 |
| 7 | R5 | ,6394085 | 21,0000003 | ,031 | ,9600636 | 1,5014869 | 23,427921 | ,0747 | 417.433,59683 | 0 |
| 8 | R6 | ,5110088 | 21,0000003 | ,031 | 1,1148078 | 2,181582 | 23,4279264 | ,0747 | 387.380,18302 | 0 |
| 9 | R7 | ,7999943 | 21,0000003 | ,031 | ,9600636 | 1,2000879 | 23,0965892 | ,1178 | 522.270,9861 | 0 |
| 10 | R8 | ,6393474 | 21,0000003 | ,031 | 1,1148078 | 1,743665 | 23,0965892 | ,1178 | 484.669,73363 | 0 |

… and export it to ***MODDE®***
(***note***: what is exported
(to a .csv-file) is the ***x**-design* and
the ***u**-formulae* for $T_{approx}^{-1}$

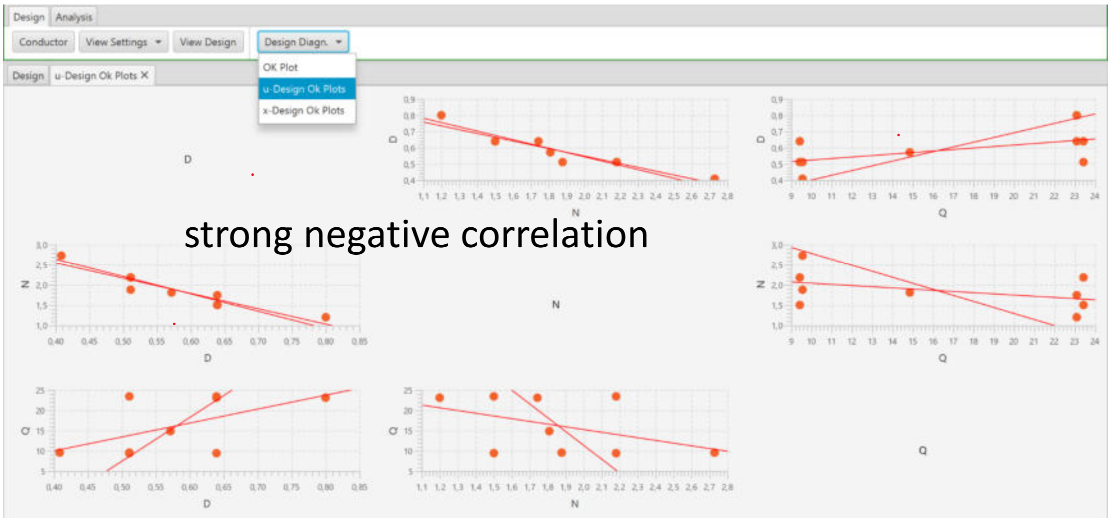


# *x-Design and u-Formulae for MODDE®*

| | vTip_x | PGV_x | OTRC_x | bMa | bMa__y |
|---|---|---|---|---|---|
| R0 | 0,01704 | -1,019 | -3,553 | 0 | 0 |
| R1 | -0,01642 | -1,12091 | -3,6902 | 0 | 0 |
| R2 | 0,05049 | -1,12091 | -3,6902 | 0 | 0 |
| R3 | -0,01642 | -0,91708 | -3,6902 | 0 | 0 |
| R4 | 0,05049 | -0,91708 | -3,6902 | 0 | 0 |
| R5 | -0,01642 | -1,12091 | -3,4158 | 0 | 0 |
| R6 | 0,05049 | -1,12091 | -3,4158 | 0 | 0 |
| R7 | -0,01642 | -0,91708 | -3,4158 | 0 | 0 |
| R8 | 0,05049 | -0,91708 | -3,4158 | 0 | 0 |

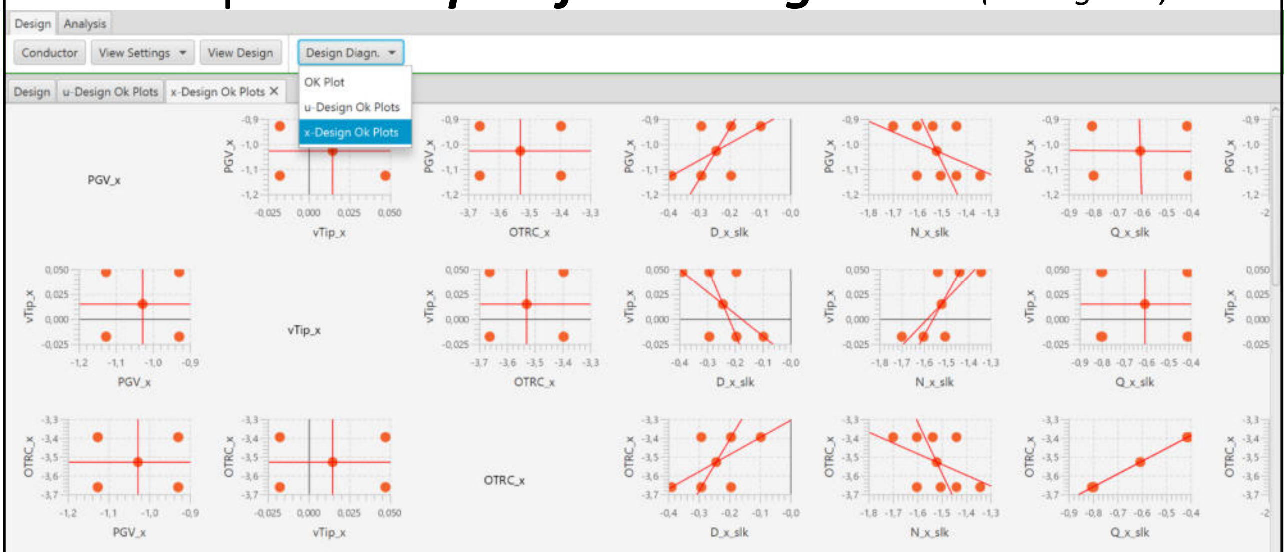
| | |
|---|---|
| D_C | (10^( - 1.5*v1+0.492178*v2+0.365707*v3+1.5790139870066067)) |
| N_C | (10^(2.5*v1 - 0.492178*v2 - 0.365707*v3 - 3.357165289403276))/( |
| Q_C | (10^(7.31414E-8*v1 - 0.0312879*v2+1.46283*v3+4.537904938837 |
| bMa_C | (10^(v6))/0.01 |
| bMa__y_C | Log10(v4*0.01) |

# Step 8ff: ***ok-plot for u-design*** *(**not** orthogonal)*

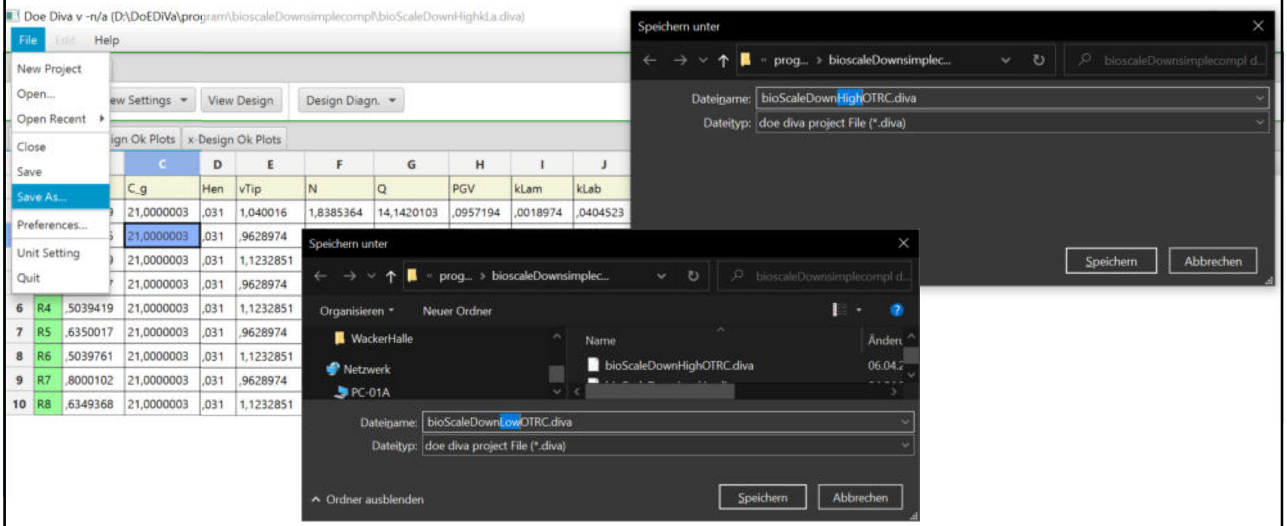





# Step 8fff: ***ok-plot for x-design*** *(orthogonal)*



# Save As „...High“ then again, this time as „...Low“






# Step 1: ***u-factors***

*(now at **low** scale)*

Factor Input | VMatrix Input | Relay Input | VMatrix | Keep

Define Factors

| Key | Name | Low | High |
|---|---|---|---|
| D | impellerDiam | 0.4 | 0.8 |
| C_g | gasVolumeConcentration | 21.0 | 21.0 |
| Hen | HenrySolubilityConst | 0.031 | 0.031 |
| vTip | tipSpeed | 1.0 | 1.0 |
| N | impellerSpeed | 1.3 | 2.6 |
| Q | gasFlowRate | 10.0 | 20.0 |
| PGV | PowerGVolumeRatio | 1.0 | 1.0 |
| kLam | oxygenTransferCoeffiMixedZone | 6.0E-4 | 0.006 |
| kLab | oxygenTransferCoeffiBubbleZone | 1.0 | 1.0 |
| OTRC | totalOxygenTransferCapacity | 1.0 | 1.0 |
| ReN | ReynoldsNumber | 1.0 | 1.0 |

Factor Input | VMatrix Input | Relay Input | VMatrix | Keep Columns | Respons

Define Factors

| Key | Name | Low | High | Role | Unit |
|---|---|---|---|---|---|
| D | impellerDiam | 0.08 | 0.16 | CONTR | m |
| C_g | gasVolumeConcentration | 21.0 | 21.0 | CONST | % |
| Hen | HenrySolubilityConst | 0.031 | 0.031 | CONST | SI |
| vTip | tipSpeed | 1.0 | 1.0 | CDEP | m/s |
| N | impellerSpeed | 6.5 | 15.0 | CONTR | revpm |
| Q | gasFlowRate | 1.0 | 2.0 | CONTR | m^3/min |
| PGV | PowerGVolumeRatio | 1.0 | 1.0 | CDEP | W/m^3 |
| kLam | oxygenTransferCoeffiMixedZone | 1.0 | 1.0 | CDEP | 1/s |
| OTRC | totalOxygenTransferCapacity | 1.0 | 1.0 | CDEP | 1/s |
| ReN | ReynoldsNumber | 1.0 | 1.0 | CDEP | SI |

No bubble zone => no factor *kLab*

# Step 1f: ***stop and think*** *(… and grumble)*

Isn't the aim of *scale down* to ***find the right settings*** to use at the low scale?

Then why do we have to ***enter these values before we start***?

Factor Input | VMatrix Input | Relay Input | VMatrix | Keep Columns | Respons

Define Factors

| Key | Name | Low | High | Role | Unit |
|---|---|---|---|---|---|
| D | impellerDiam | 0.08 | 0.16 | CONTR | m |
| C_g | gasVolumeConcentration | 21.0 | 21.0 | CONST | % |
| Hen | HenrySolubilityConst | 0.031 | 0.031 | CONST | SI |
| vTip | tipSpeed | 1.0 | 1.0 | CDEP | m/s |
| N | impellerSpeed | 6.5 | 15.0 | CONTR | revpm |
| Q | gasFlowRate | 1.0 | 2.0 | CONTR | m^3/min |
| PGV | PowerGVolumeRatio | 1.0 | 1.0 | CDEP | W/m^3 |
| kLam | oxygenTransferCoeffiMixedZone | 1.0 | 1.0 | CDEP | 1/s |
| OTRC | totalOxygenTransferCapacity | 1.0 | 1.0 | CDEP | 1/s |
| ReN | ReynoldsNumber | 1.0 | 1.0 | CDEP | SI |



# Step 1ff: ***don't worry,*** *(don't grumble)* ***… be happy***

The *„All that one knows"*, *A-t-o-k,* approach requires you to use *A-t-y-k ☺.*

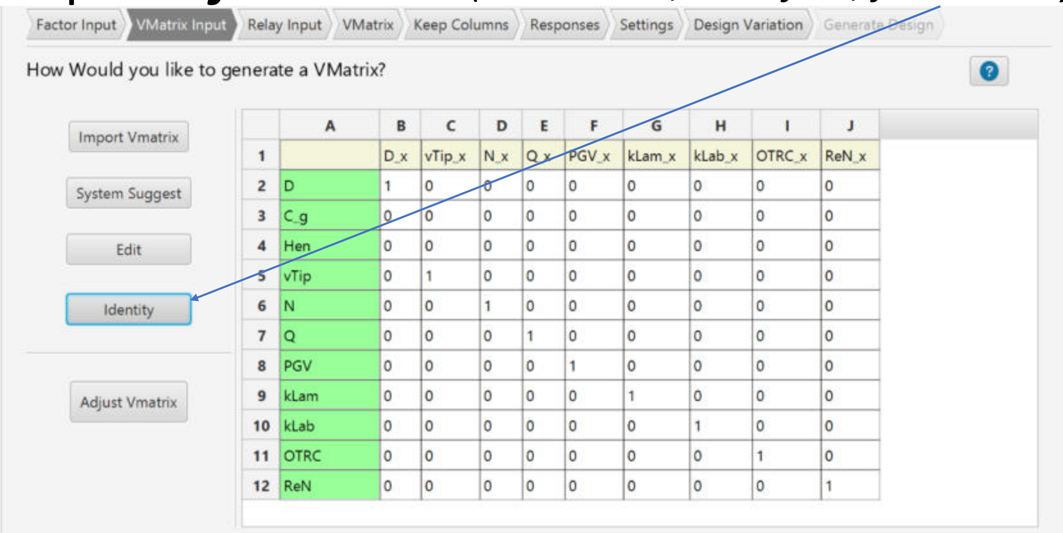
The Limits are only important to ***define the scope of validity*** of the *formulae (slide after next slide).*

Factor Input | VMatrix Input | Relay Input | VMatrix | Keep Columns | Respons

Define Factors

| Key | Name | Low | High | Role | Unit |
|---|---|---|---|---|---|
| D | impellerDiam | 0.08 | 0.16 | CONTR | m |
| C_g | gasVolumeConcentration | 21.0 | 21.0 | CONST | % |
| Hen | HenrySolubilityConst | 0.031 | 0.031 | CONST | SI |
| vTip | tipSpeed | 1.0 | 1.0 | CDEP | m/s |
| N | impellerSpeed | 6.5 | 15.0 | CONTR | revpm |
| Q | gasFlowRate | 1.0 | 2.0 | CONTR | m^3/min |
| PGV | PowerGVolumeRatio | 1.0 | 1.0 | CDEP | W/m^3 |
| kLam | oxygenTransferCoeffiMixedZone | 1.0 | 1.0 | CDEP | 1/s |
| OTRC | totalOxygenTransferCapacity | 1.0 | 1.0 | CDEP | 1/s |
| ReN | ReynoldsNumber | 1.0 | 1.0 | CDEP | SI |

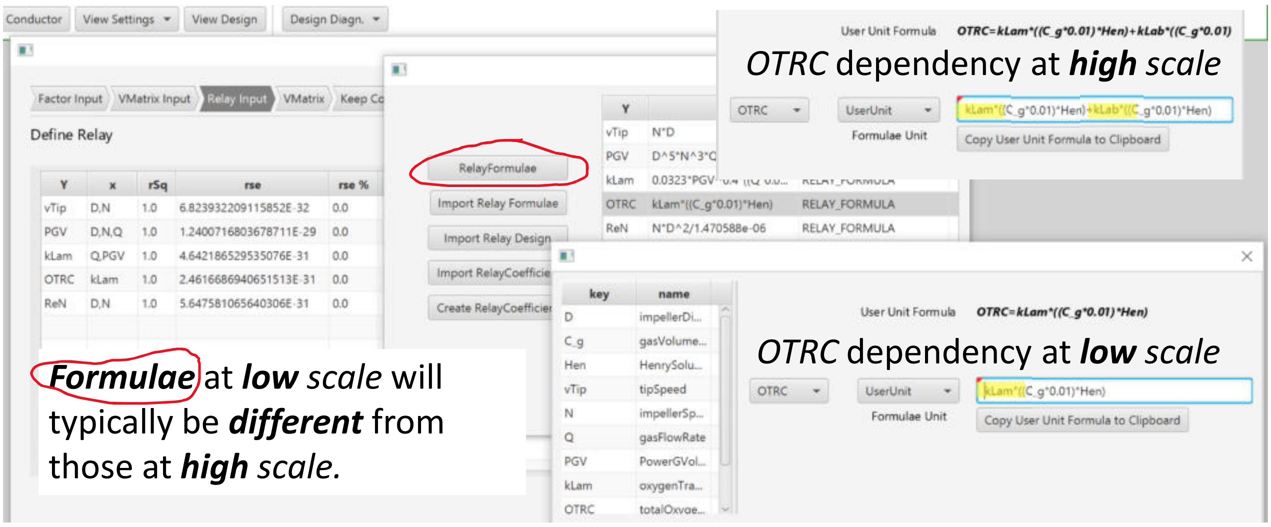
# Step 2: ***x-factors*** *(in this case, as before, just identity)*




| | A | B | C | D | E | F | G | H | I | J |
|---|---|---|---|---|---|---|---|---|---|---|
| 1 | | D_x | vTip_x | N_x | Q_x | PGV_x | kLam_x | kLab_x | OTRC_x | ReN_x |
| 2 | D | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 3 | C_g | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 4 | Hen | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 5 | vTip | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 6 | N | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 7 | Q | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 | 0 |
| 8 | PGV | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 | 0 |
| 9 | kLam | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 | 0 |
| 10 | kLab | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 | 0 |
| 11 | OTRC | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| 12 | ReN | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |



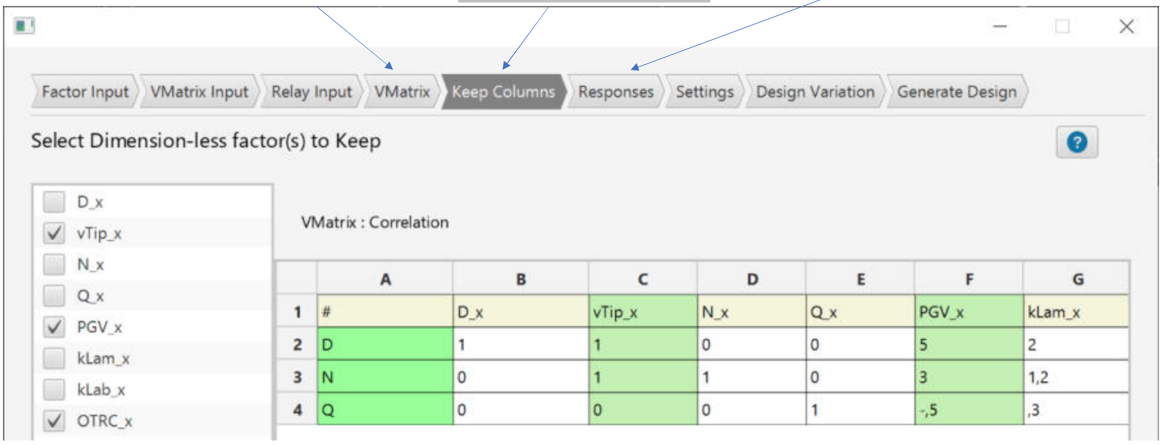
# Step 3: ***u-factor dependencies*** *(example OTRC)*




***Formulae*** at ***low*** *scale* will typically be ***different*** from those at ***high*** *scale.*

# Steps 4, 5 & 6: are the *same* as at *high Scale*

*(Vmatrix-Info, Keep Columns and Responses)*




| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | # | D_x | vTip_x | N_x | Q_x | PGV_x | kLam_x |
| 2 | D | 1 | 1 | 0 | 0 | 5 | 2 |
| 3 | N | 0 | 1 | 1 | 0 | 3 | 1,2 |
| 4 | Q | 0 | 0 | 0 | 1 | -,5 | ,3 |



# Step 7: ***x-settings*** *should be comparable …*

High scale | Low scale

| High: User Low | High: Inner Low | High: Mean | High: Inner High | High: User High | # | Low: User Low | Low: Inner Low | Low: Mean | Low: Inner High | Low: User High |
|---|---|---|---|---|---|---|---|---|---|---|
| 0.96291 | 0.96291 | 1.04 | 1.12326 | 1.12326 | vTip_x | 0.986631 | 0.986631 | 1.11714 | 1.26491 | 1.26491 |
| 0.075692 | 0.075692 | 0.0957172 | 0.121041 | 0.121041 | PGV_x | 0.0121753 | 0.0121753 | 0.0150062 | 0.0184957 | 0.0184957 |
| 2.04085E-4 | 2.04085E-4 | 2.79898E-4 | 3.83875E-4 | 3.83875E-4 | OTRC_x | 1.28908E-5 | 1.28908E-5 | 1.41E-5 | 1.54227E-5 | 1.54227E-5 |
| 0.565685 | 0.565685 | 0.565685 | 0.565685 | 0.565685 | D_N | 0.113137 | 0.113137 | 0.113137 | 0.113137 | 0.113137 |
| 21.0 | 21.0 | 21.0 | 21.0 | 21.0 | C_g | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 |

*Inner* ***Low*** for *PGV* at *high scale* is ***below*** ☹ *Inner* ***High*** at *low scale*
and also
*Inner* ***Low*** for *OTRC at high scale is* ***above*** ☹ *Inner* ***High*** *at low scale*

## Step 7f: … *otherwise check* ***x-settings / VRES***

If they are „too different“, use infos in *VRES* ($T_{approx}$) and *WRES* ($T_{approx}^{-1}$) to get indications on which ***u-factor limits*** or *dependency formulae* should be changed

Vmatrix | VRes | Wres | y-response(s)

Low scale

| # | User Low | Inner Low | Mean | Inner High | User High |
|---|---|---|---|---|---|
| vTip_x | 0.986631 | 0.986631 | 1.11714 | 1.26491 | 1.26491 |
| PGV_x | 0.0121753 | 0.0121753 | 0.0150062 | 0.0184957 | 0.0184957 |
| OTRC_x | 1.28908E-5 | 1.28908E-5 | 1.41E-5 | 1.54227E-5 | 1.54227E-5 |
| D_N | 0.113137 | 0.113137 | 0.113137 | 0.113137 | 0.113137 |
| C_g | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 |
| Hen | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 |
| vTip_D | 1.11714 | 1.11714 | 1.11714 | 1.11714 | 1.11714 |
| N_N | 9.87421 | 9.87421 | 9.87421 | 9.87421 | 9.87421 |

Use *VRES* (*i.e.* $T_{approx}$) *Information to lift* ***PGV*** and ***OTRC*** at *low scale*



## Step 7ff: Use *VRES to lift* ***PGV*** and ***OTRC***

Vmatrix | x-Settings | u-Settings | VRes | Wres | y-response(s)

| | A | B | C | D | E | F | G |
|---|---|---|---|---|---|---|---|
| 1 | | vTip_x | PGV_x | OTRC_x | D_N | C_g | Hen |
| 2 | D | 1 | 5 | 2 | 0 | 0 | 0 |
| 3 | D_N | -1 | -5 | -2 | 1 | 0 | 0 |
| 4 | C_g | 0 | 0 | 0 | 0 | 1 | 0 |
| 5 | Hen | 0 | 0 | 0 | 0 | 0 | 1 |
| 6 | vTip_D | 1 | 0 | 0 | 0 | 0 | 0 |
| 7 | N | 1 | 3 | 1,2 | 0 | 0 | 0 |
| 8 | N_N | -1 | -3 | -1,2 | 0 | 0 | 0 |
| 9 | Q | 0 | -,5 | ,3 | 0 | 0 | 0 |

***D*** ↗, ***N*** ↗, ***Q*** ↘ to improve ***PGV***,
***D*** ↗, ***N*** ↗, ***Q*** ↗ to improve ***OTRC***

Define Factors

*try this*

| Key | Name | Low | High | Low (try this) | High (try this) |
|---|---|---|---|---|---|
| D | impellerDiam | 0.08 | 0.16 | 0.12 | 0.2 |
| C_g | gasVolumeConcent | 21.0 | 21.0 | 21.0 | 21.0 |
| Hen | HenrySolubilityCon: | 0.031 | 0.031 | 0.031 | 0.031 |
| vTip | tipSpeed | 1.0 | 1.0 | 1.0 | 1.0 |
| N | impellerSpeed | 6.5 | 15.0 | 6.5 | 15.0 |
| Q | gasFlowRate | 1.0 | 2.0 | 1.0 | 2.0 |
| PGV | PowerGVolumeRati | 1.0 | 1.0 | 1.0 | 1.0 |
| kLam | oxygenTransferCoe | 1.0 | 1.0 | 1.0 | 1.0 |
| OTRC | totalOxygenTransfe | 1.0 | 1.0 | 1.0 | 1.0 |
| ReN | ReynoldsNumber | 1.0 | 1.0 | 1.0 | 1.0 |

## Step 7fff: ***x-settings*** *should not be too different*

High scale

| # | User Low | Inner Low | Mean | Inner High | User High |
|---|---|---|---|---|---|
| vTip_x | 0.96291 | 0.96291 | 1.04 | 1.12326 | 1.12326 |
| PGV_x | 0.075692 | 0.075692 | 0.0957172 | 0.121041 | 0.121041 |
| OTRC_x | 2.04085E-4 | 2.04085E-4 | 2.79898E-4 | 3.83875E-4 | 3.83875E-4 |
| D_N | 0.565685 | 0.565685 | 0.565685 | 0.565685 | 0.565685 |
| C_g | 21.0 | 21.0 | 21.0 | 21.0 | 21.0 |
| Hen | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 |
| vTip_D | 1.04 | 1.04 | 1.04 | 1.04 | 1.04 |
| N_N | 1.83849 | 1.83849 | 1.83849 | 1.83849 | 1.83849 |

Adapted Low scale

| User Low | Inner Low | Mean | Inner High | User High |
|---|---|---|---|---|
| 1.38535 | 1.38535 | 1.52971 | 1.6891 | 1.6891 |
| 0.0590908 | 0.0590908 | 0.0722404 | 0.0883141 | 0.0883141 |
| 2.41485E-5 | 2.41485E-5 | 2.65003E-5 | 2.90804E-5 | 2.90804E-5 |
| 0.154919 | 0.154919 | 0.154919 | 0.154919 | 0.154919 |
| 21.0 | 21.0 | 21.0 | 21.0 | 21.0 |
| 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 | 0.0309999 |
| 1.52971 | 1.52971 | 1.52971 | 1.52971 | 1.52971 |
| 9.87421 | 9.87421 | 9.87421 | 9.87421 | 9.87421 |

*PGV* is **better**, *vTip* may just be ***good enough***, *OTRC* is ***definitely not***



## Step 7ffff: Use *Oxygen instead of air to lift* ***OTRC***

Define Factors

| Low | High | Key | Name | Low | High |
|---|---|---|---|---|---|
| 0.12 | 0.2 | D | impellerDiam | 0.12 | 0.2 |
| 21.0 | 21.0 | C_g | gasVolumeConcentra | 100.0 | 100.0 |
| 0.031 | 0.031 | Hen | HenrySolubilityConst | 0.031 | 0.031 |
| 1.0 | 1.0 | vTip | tipSpeed | 1.0 | 1.0 |
| 6.5 | 15.0 | N | impellerSpeed | 6.5 | 13.0 |
| 1.0 | 2.0 | Q | gasFlowRate | 1.0 | 2.0 |
| 1.0 | 1.0 | PGV | PowerGVolumeRatio | 1.0 | 1.0 |
| 1.0 | 1.0 | kLam | oxygenTransferCoeff | 1.0 | 1.0 |
| 1.0 | 1.0 | OTRC | totalOxygenTransfer( | 1.0 | 1.0 |
| 1.0 | 1.0 | ReN | ReynoldsNumber | 1.0 | 1.0 |

High scale

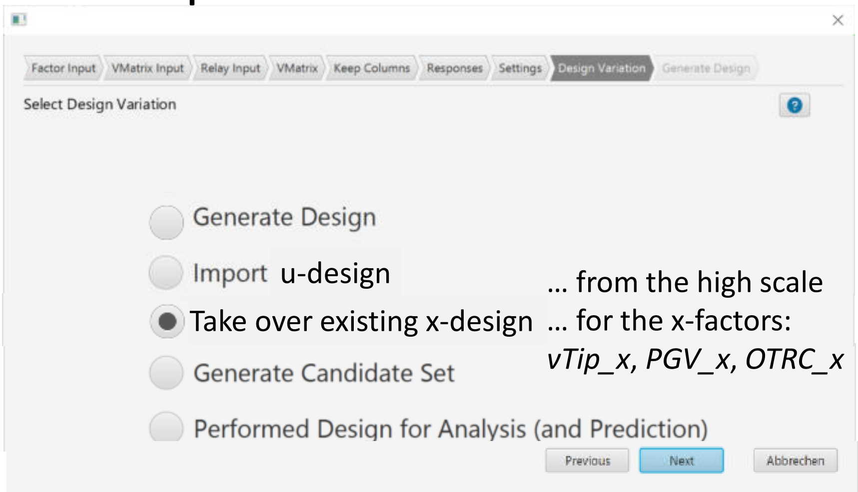
| User Low | Inner Low | Mean | Inner High | User High |
|---|---|---|---|---|
| 0.96291 | 0.96291 | 1.04 | 1.12326 | 1.12326 |
| 0.075692 | 0.075692 | 0.0957172 | 0.121041 | 0.121041 |
| 2.04085E-4 | 2.04085E-4 | 2.79898E-4 | 3.83875E-4 | 3.83875E-4 |
| 0.565685 | 0.565685 | 0.565685 | 0.565685 | 0.565685 |

Re-Adapted Low scale

| User Low | Inner Low | Mean | Inner High | User High |
|---|---|---|---|---|
| 1.38535 | 1.38535 | 1.52971 | 1.6891 | 1.6891 |
| 0.0590908 | 0.0590908 | 0.0722404 | 0.0883141 | 0.0883141 |
| 1.15093E-4 | 1.15093E-4 | 1.26299E-4 | 1.38599E-4 | 1.38599E-4 |
| 0.154919 | 0.154919 | 0.154919 | 0.154919 | 0.154919 |

*PGV* and *vTip* are **unchanged**, *OTRC* is ***now*** $^{1}/_{2}$ ***and not*** $^{1}/_{10}$ of *high scale*

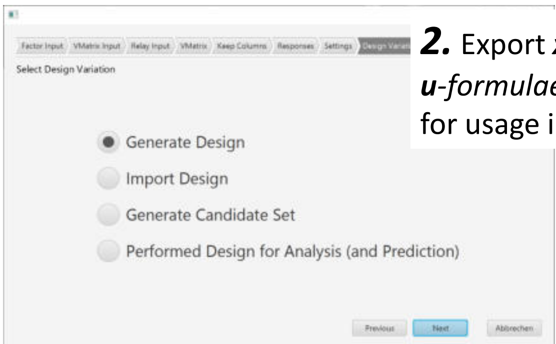
# Next Step: ***Future DoE-DiVa Functionality***



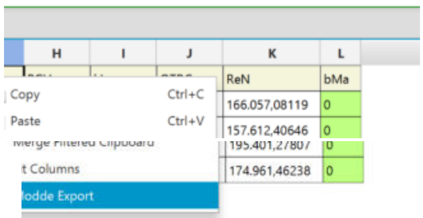



# Step 8: ***Current DoE-DiVa Functionality***

***1.*** Generate new ***x****-design* at *low scale* for *vTip_x, PGV_x, OTRC_x*




***2.*** Export ***x****-design* and ***u****-formulae* to .csv for usage in ***MODDE®***




***3.*** In ***MODDE®*** use the ***x****-design* from high scale, ***u****-formulae* from *low scale*

| | |
|---|---|
| D_C | (10^( - 1.5*v1+0.3*v2+0.5*v3+2.09750160048793)) |
| N_C | (10^(2.5*v1 - 0.3*v2 - 0.5*v3 - 3.875652761201205)), |
| Q_C | (10^( - 0.8*v2+2*v3+6.611854831785227))/0.016666 |
| bMa_C | (10^(v6))/0.01 |
| bMa__y_C | Log10(v4*0.01) |

# Step 8f: ***u-Formulae*** *at the two Scales* ***for MODDE®***

| | |
|---|---|
| D_C | (10^( - 1.5*v1+0.492178*v2+0.365707*v3+1.5790139870066067)) |
| N_C | (10^(2.5*v1 - 0.492178*v2 - 0.365707*v3 - 3.357165289403276))/( |
| Q_C | (10^(7.31414E-8*v1 - 0.0312879*v2+1.46283*v3+4.537904938837 |
| bMa_C | (10^(v6))/0.01 |
| bMa__y_C | Log10(v4*0.01) |

***High scale***

*$T_y$ and $T_y^{-1}$*

***$T_{approx}^{-1}$ at high Scale***

Please note: they are *different*!

***Low scale***

| | |
|---|---|
| D_C | (10^( - 1.5*v1+0.3*v2+0.5*v3+1.7586825108558148)) |
| N_C | (10^(2.5*v1 - 0.3*v2 - 0.5*v3 - 3.5368338238484998))/0.01666666 |
| Q_C | (10^( - 0.8*v2+2*v3+5.256579707449209))/0.0166666666667 |
| bMa_C | (10^(v6))/0.01 |
| bMa__y_C | Log10(v4*0.01 |

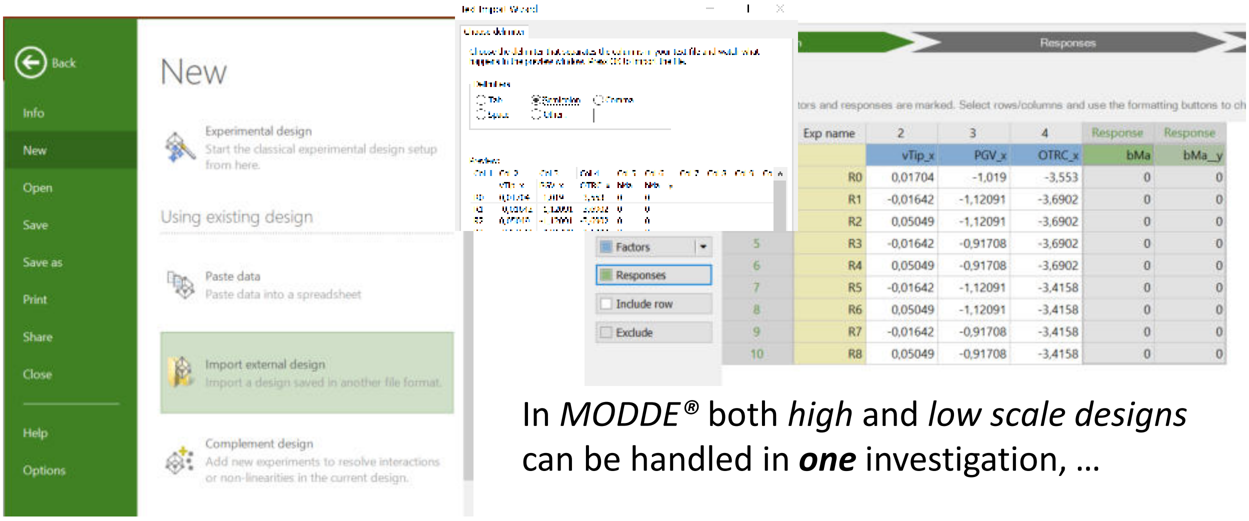
*$T_y$ and $T_y^{-1}$*

***$T_{approx}^{-1}$ at low Scale***



1. The DoE-Diva approach
2. Bioreactor at High and Low Scale
3. Preparing Scale Down using DoE-DiVa
4. ***Performing Scale Down using MODDE®***

# Step 9: Import *High Scale x-design* into *MODDE®*



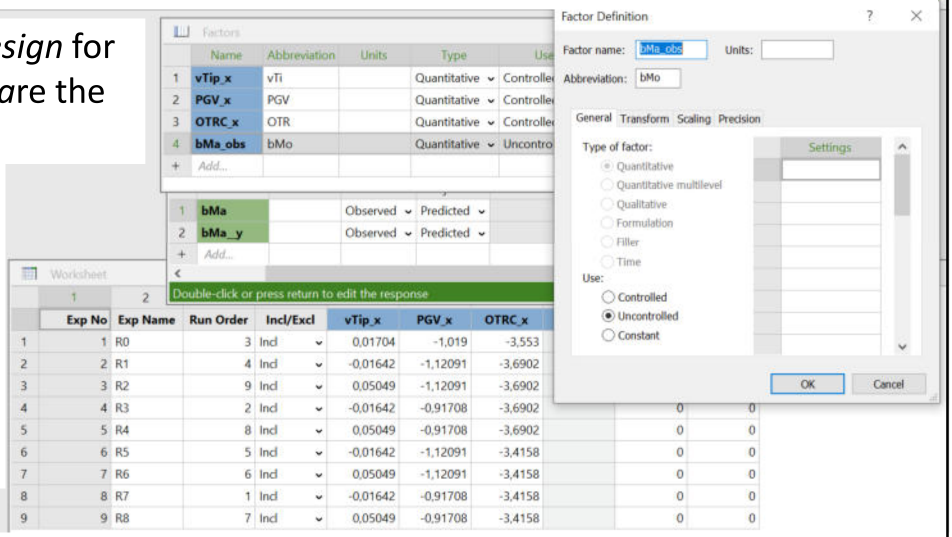

In *MODDE®* both *high* and *low scale designs* can be handled in ***one*** investigation, …



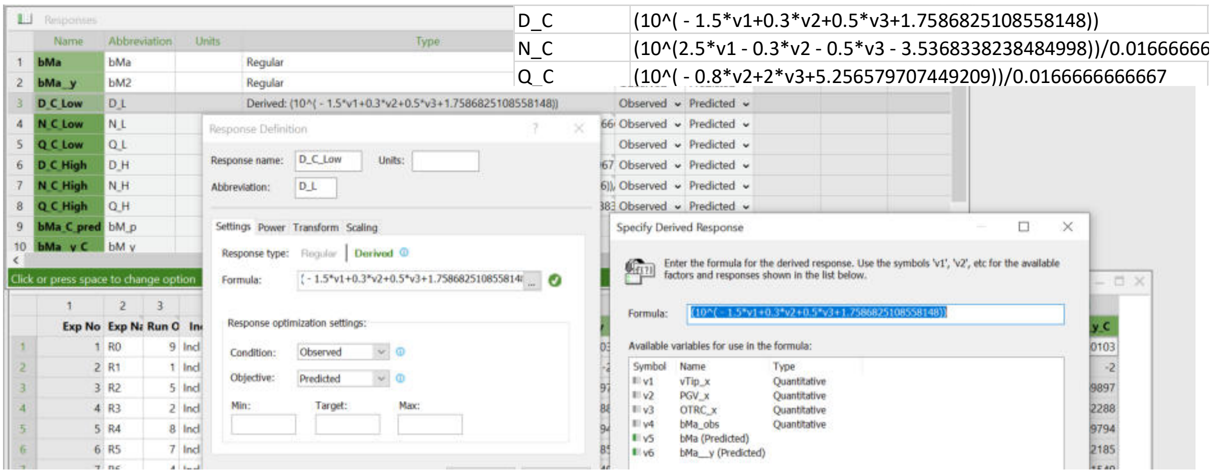
# Step 9f: Import *High Scale x-design, set z-response*

… because the ***x****-design* for *low* and *high scale* are the same.

***Don't forget***: For each ***z****-response (i.e. measured response) we need an uncontrolled factor (here bio-Mass, **bMa_obs**)*



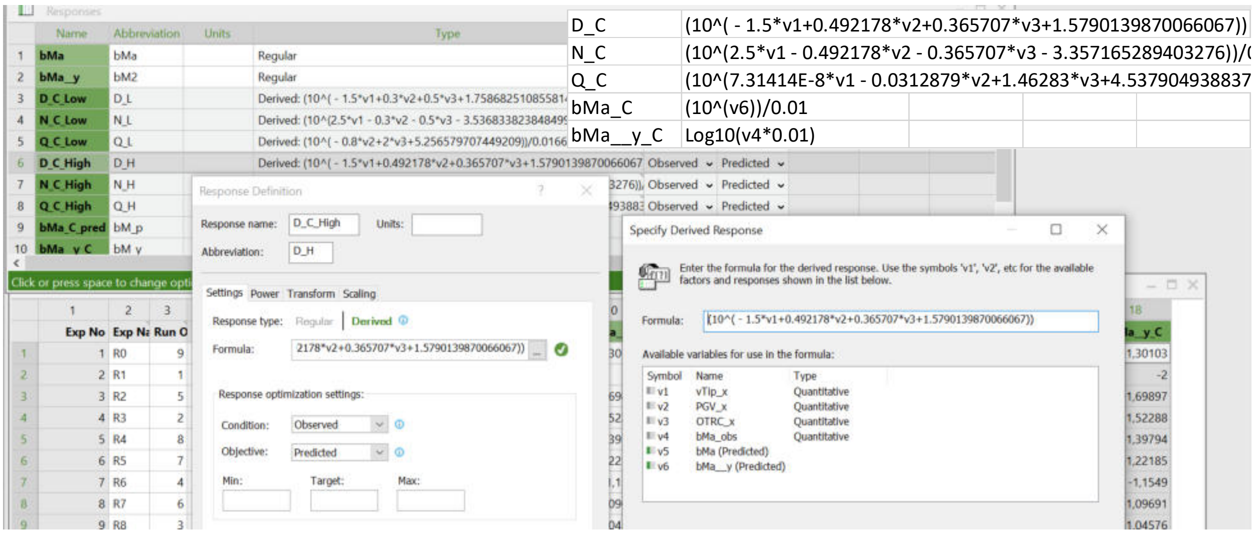
# Step 10: Copy *Low* ***Scale*** *formulae* to ***MODDE®***




***Note:*** The <u>formulae for the ***u****-factor* settings</u> are different at *low* and *high* scales!



# Step 10f: Copy *High* ***Scale*** *formulae* to ***MODDE®***



# Step 10ff: ***MODDE®*** *automatically does* ***Scale Down***

| | | | | | | |
|---|---|---|---|---|---|---|
| 1 | bMa | bMa | | Regular | Desired | Maximize |
| 2 | bMa_y | bM2 | | Regular | Observed | Predicted |
| 3 | D_C_Low | D_L | | Derived: (10^( - 1.5*v1+0.3*v2+0.5*v3+1.7586825108558148)) | Observed | Predicted |
| 4 | N_C_Low | N_L | | Derived: (10^(2.5*v1 - 0.3*v2 - 0.5*v3 - 3.5368338238484998))/0.0166666 | Observed | Predicted |
| 5 | Q_C_Low | Q_L | | Derived: (10^( - 0.8*v2+2*v3+5.256579707449209))/0.0166666666667 | Observed | Predicted |
| 6 | D_C_High | D_H | | Derived: (10^( - 1.5*v1+0.492178*v2+0.365707*v3+1.5790139870066067 | Observed | Predicted |
| 7 | N_C_High | N_H | | Derived: (10^(2.5*v1 - 0.492178*v2 - 0.365707*v3 - 3.357165289403276)), | Observed | Predicted |
| 8 | Q_C_High | Q_H | | Derived: (10^(7.31414E-8*v1 - 0.0312879*v2+1.46283*v3+4.53790493883 | Observed | Predicted |
| 9 | bMa_C_pred | bM_p | | Derived: (10^(v6))/0.01 | Observed | Predicted |
| 10 | bMa_y_C | bM_y | | Derived: Log10(v4*0.01) | Observed | Predicted |

Click or press space to change option

Design to perform at *low scale*

Equivalent Design at *high scale*

| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Exp No | Exp Na | Run O | Incl/Excl | vTip_x | PGV_x | OTRC_x | bMa_obs | bMa | bMa_y | D_C_Low | N_C_Low | Q_C_Low | D_C_High | N_C_High | Q_C_High | bMa_C_pred | bMa_y_C |
| 1 | 1 | R0 | 9 | Incl | 0,01704 | -1,019 | -3,553 | 5 | 5 | -1,30103 | 0,447633 | 2,32336 | 5,54538 | 0,565676 | 1,83854 | 14,1418 | | -1,30103 |
| 2 | 2 | R1 | 1 | Incl | -0,01642 | -1,12091 | -3,6902 | 1 | 1 | -2 | 0,399889 | 2,40791 | 3,5568 | 0,503993 | 1,91054 | 8,97412 | | -2 |
| 3 | | | | | | | | | | -1,69897 | 0,317376 | 3,53929 | 3,5568 | 0,4 | 2,80821 | 8,97412 | | -1,69897 |
| 4 | | | | | | | | | | -1,52288 | 0,46035 | 2,09166 | 2,4434 | 0,634959 | 1,51647 | 8,8433 | | -1,52288 |
| 5 | | | | | | | | | | -1,39794 | 0,365362 | 3,07444 | 2,4434 | 0,503942 | 2,229 | 8,8433 | | -1,39794 |
| 6 | | | | | | | | | | -1,22185 | 0,548453 | 1,75566 | 12,5852 | 0,635002 | 1,51637 | 22,6148 | | -1,22185 |
| 7 | | | | | | | | | | -1,1549 | 0,435286 | 2,58057 | 12,5852 | 0,503976 | 2,22885 | 22,6148 | | -1,1549 |
| 8 | | | | | | | | | | -1,09691 | 0,631377 | 1,52508 | 8,6456 | 0,80001 | 1,20361 | 22,2851 | | -1,09691 |
| 9 | | | | | | | | | | -1,04576 | 0,501099 | 2,24164 | 8,6456 | 0,634937 | 1,76913 | 22,2851 | | -1,04576 |

**Low** **High**

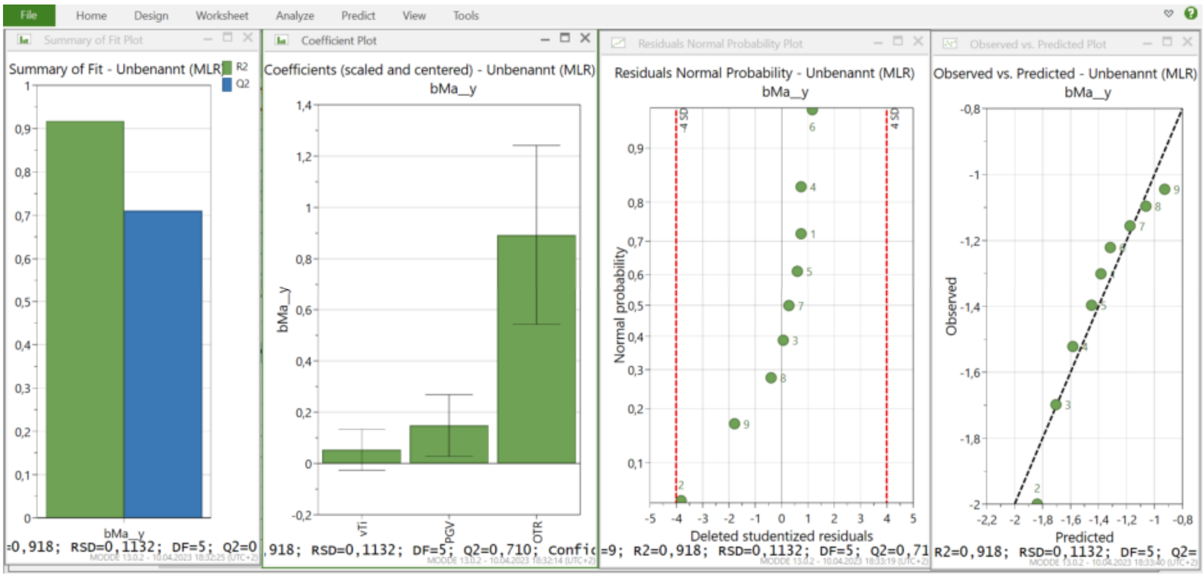
Disappointing?

*Scale down factor < 1.5*

*Well, PGV ~ $d^5$ and $1{,}5^5 = 7.6$*



# Step 11: *Analysis* in ***MODDE®*** for ***x-factor model***



# Step 12: Optimization in ***MODDE®***

File Home Design Worksheet Analyze Predict View Tools Optimizer

Optimizer | Coefficient Plot | Residuals Normal Probability Plot | Observed vs. Predicted Plot | Summary of Fit Plot | Worksheet | Responses

**Objective**

To get started:

1. Check response settings
   Target D_C_Low at 0,1
   Target D_C_High at 0,5
2. Check factor settings
3. Click 'Run optimizer'

Run optimizer

Objective | Setpoint | Alternative setpoints

| # | Name | Condition | Objective | Min | Target | Max | Predicted min | Predicted max | Response range |
|---|---|---|---|---|---|---|---|---|---|
| 1 | bMa | Observed | Predicted | | | | 15,7953 | 33,879 | |
| 2 | bMa_y | Observed | Predicted | | | | -0,315332 | 1,65014 | |
| 3 | D_C_Low | Required | Target | 0,09 | 0,1 | 0,11 | 0,317378 | 0,589408 | |
| 4 | N_C_Low | Observed | Predicted | | | | 1,65903 | 3,53926 | |
| 5 | Q_C_Low | Observed | Predicted | | | | 2,4434 | 10,6826 | |
| 6 | D_C_High | Required | Target | 0,49 | 0,5 | 0,51 | 0,4 | 0,74643 | |
| 7 | N_C_High | Observed | Predicted | | | | 1,31003 | 2,80821 | |
| 8 | Q_C_High | Observed | Predicted | | | | 8,84329 | 20,588 | |
| 9 | bMa_C_pred | Desired | Maximize | 48,3952 | 4468,26 | | 48,3952 | 4468,26 | |
| 10 | bMa_y_C | Observed | Predicted | | | | -2 | -1,04576 | |

| # | Factor | Role | Value | Low limit | High limit | Precision | Factor range |
|---|---|---|---|---|---|---|---|
| 1 | vTip_x | Free | | -0,01642 | 0,05049 | | |
| 2 | PGV_x | Free | | -1,12091 | -0,91708 | | |
| 3 | OTRC_x | Free | | -3,6902 | -3,4158 | | |
| 4 | bMa_obs | Free | | 1 | 9 | | |

Aim at ***D = 0.1*** at low scale, and ***D = 0,5*** at high scale



# Step 12f: Optimization in ***MODDE®***

File Home Design Worksheet Analyze Predict View Tools Optimizer

Optimizer | Coefficient Plot | Residuals Normal Probability Plot | Observed vs. Predicted Plot | Summary of Fit Plot | Worksheet | Responses

**Setpoint**

Selected setpoint: **#9**

Alternative setpoints:

| # | log(D) | Prob. of failure |
|---|---|---|
| 5 | -0,01 | |
| 6 | -0,00641 | |
| 7 | -0,0127 | |
| 8 | -0,0121 | |
| 9 | -0,0137 | |

Select best run

Find robust setpoint

The run with lowest log(D) is selected.

Objective | Setpoint (#9) | Alternative setpoints

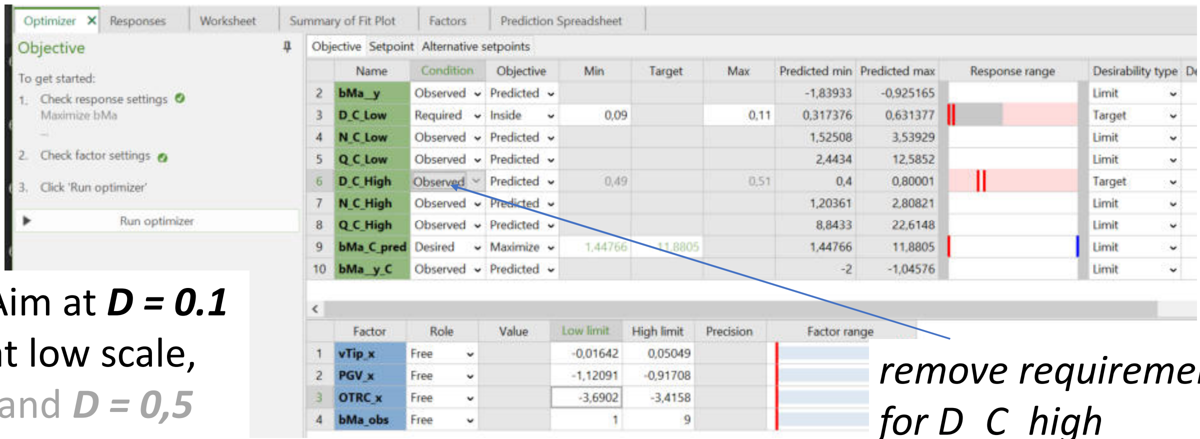
| # | Response | Objective | Value | Response range | log(D) | Prob. of failure | Cpk |
|---|---|---|---|---|---|---|---|
| 1 | bMa | Predicted | 18,8395 | | | | |
| 2 | bMa_y | Predicted | 0,0709357 | | | | |
| 3 | D_C_Low | Inside | 0,360567 | | | | |
| 4 | N_C_Low | Predicted | 3,112 | | | | |
| 5 | Q_C_Low | Predicted | 2,62661 | | | | |
| 6 | D_C_High | Inside | 0,490431 | | | 0 % | |
| 7 | N_C_High | Predicted | 2,28796 | | | | |
| 8 | Q_C_High | Predicted | 8,9899 | | | | |
| 9 | bMa_C_pred | Maximize | 117,748 | | -0,0137373 | | |
| 10 | bMa_y_C | Predicted | -1,18289 | | | | |

***far away*** (D_C_Low)

***ok*** (D_C_High)

| # | Factor | Role | Value | Factor range | Factor contribution |
|---|---|---|---|---|---|
| 1 | vTip_x | Free | 0,0500243 | | 33,3397 |
| 2 | PGV_x | Free | -0,945655 | | 33,3255 |
| 3 | OTRC_x | Free | -3,68593 | | 33,3348 |
| 4 | bMa_obs | Free | 6,56315 | | 0 |

# Step 12ff: Change specification – desirabilities

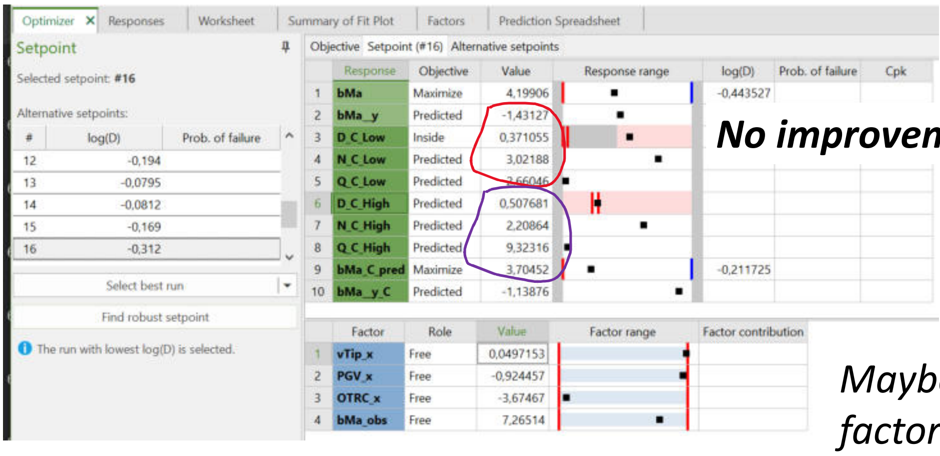



Aim at ***D = 0.1*** at low scale,
(and ***D = 0,5*** at high scale)

*remove requirement for D_C_high*



# Step 12fff: 2nd Optimization in ***MODDE®***




***No improvement***

*Maybe widen factor intervals*

# Step 12ffff: Widen factor intervals

Worksheet | Summary of Fit Plot | Factors | Prediction Spreadsheet

Objective | Setpoint | Alternative setpoints

| | Name | Condition | Objective | Min | Target | Max | Predicted min | Predicted max | Response range |
|---|---|---|---|---|---|---|---|---|---|
| 1 | bMa | Desired | Maximize | -2,93762 | 11,0031 | | -2,93762 | 11,0031 | |
| 2 | bMa_y | Observed | Predicted | | | | -2,25595 | -0,652752 | |
| 3 | D_C_Low | Required | Inside | 0,09 | | 0,11 | 0,156698 | 0,631377 | |
| 4 | N_C_Low | Observed | Predicted | | | | 1,52508 | 9,32049 | |
| 5 | Q_C_Low | Observed | Predicted | | | | 0,929962 | 13,0356 | |
| 6 | D_C_High | Observed | Predicted | 0,49 | | 0,51 | 0,2069 | 0,80001 | |
| 7 | N_C_High | Observed | Predicted | | | | 1,20361 | 7,03739 | |
| 8 | Q_C_High | Observed | Predicted | | | | 4,36403 | 22,6459 | |
| 9 | bMa_C_pred | Desired | Maximize | 0,552278 | 22,2391 | | 0,552278 | 22,2391 | |
| 10 | bMa_y_C | Observed | Predicted | | | | -2 | -1,04576 | |

| | Factor | Role | Value | Low limit | High limit | Precision | Factor range |
|---|---|---|---|---|---|---|---|
| 1 | vTip_x | Free | | -0,01642 | 0,1846 | | |
| 2 | PGV_x | Free | | -1,14 | -0,91708 | | |
| 3 | OTRC_x | Free | | -3,9 | -3,4158 | | |
| 4 | bMa_obs | Free | | 1 | 9 | | |

*Changed factor intervals*

**Transformation**
- (●) LOG
- ( ) back-transform

| # | Inner Low | Mean | Inner High |
|---|---|---|---|
| vTip_x | 0.141561 | 0.184608 | 0.227655 |
| PGV_x | -1.22848 | -1.14122 | -1.05397 |
| OTRC_x | -3.93895 | -3.8986 | -3.85824 |
| D_N | -0.809894 | -0.809894 | -0.809894 |

Re-Adapted *Low scale* from slide 50

| # | Inner Low | Mean | Inner High |
|---|---|---|---|
| vTip_x | 1.38535 | 1.52971 | 1.6891 |
| PGV_x | 0.0590908 | 0.0722404 | 0.0883141 |
| OTRC_x | 1.15093E-4 | 1.26299E-4 | 1.38599E-4 |
| [illegible] | 0.154919 | 0.154919 | 0.154919 |

**Transformation**
- ( ) LOG
- (●) back-transform



# Step 12fffff: 3rd Optimization in ***MODDE®***

Optimizer | Responses | Worksheet | Summary of Fit Plot | Factors | Prediction Spreadsheet

**Setpoint**

Selected setpoint: **#16**

Alternative setpoints:

| # | log(D) | Prob. of ... |
|---|---|---|
| 12 | -0,177 | |
| 13 | -0,111 | |
| 14 | -0,183 | |
| 15 | -0,17 | |
| 16 | -0,204 | |

Select best run

Find robust setp[oint] — Select the run with lowest log(D) or Prob. fail. (%). Click the arrow for options.

The run with lowest log(D) is selected.

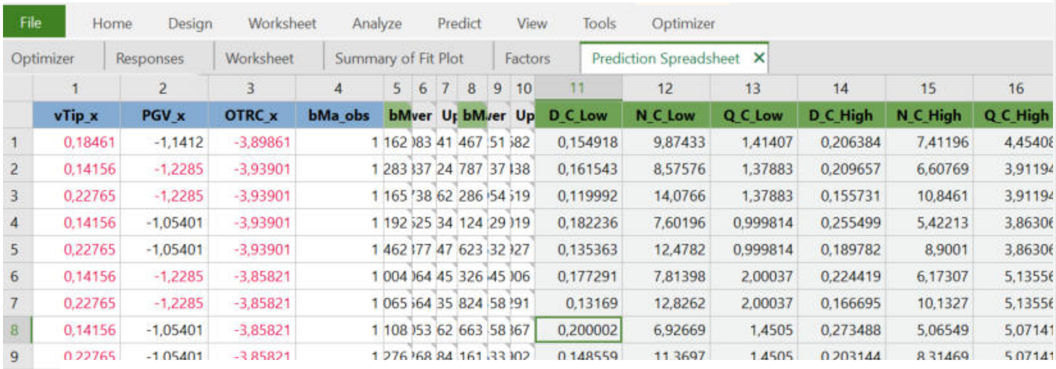
Objective | Setpoint (#16) | Alternative setpoints

| | Response | Objective | Value | Response range | log(D) | Prob. of failure | Cpk |
|---|---|---|---|---|---|---|---|
| 1 | bMa | Maximize | 1,96281 | | -0,376206 | | |
| 2 | bMa_y | Predicted | -1,60776 | | | | |
| 3 | D_C_Low | Inside | 0,178566 | | | | |
| 4 | N_C_Low | Predicted | 8,54685 | | | | |
| 5 | Q_C_Low | Predicted | 0,982821 | | | | |
| 6 | D_C_High | Predicted | 0,259768 | | | | |
| 7 | N_C_High | Predicted | 5,87514 | | | | |
| 8 | Q_C_High | Predicted | 4,40167 | | | | |
| 9 | bMa_C_pred | Maximize | 2,46748 | | -0,0803073 | | |
| 10 | bMa_y_C | Predicted | -1,0843 | | | | |

| | Factor | Role | Value | Factor range | Factor contribution |
|---|---|---|---|---|---|
| 1 | vTip_x | Free | 0,183604 | | |
| 2 | PGV_x | Free | -0,941829 | | |
| 3 | OTRC_x | Free | -3,89786 | | |
| 4 | bMa_obs | Free | 8,23573 | | |

*D_C_Low is now better*
*but*
*Scale-up factor is only 1,4*

# Step 13: Design validation in ***MODDE®***




| | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | vTip_x | PGV_x | OTRC_x | bMa_obs | bMver | Up | bMver | Up | | | D_C_Low | N_C_Low | Q_C_Low | D_C_High | N_C_High | Q_C_High |
| 1 | 0,18461 | -1,1412 | -3,89861 | 1 | 162 | 83 | 41 | 467 | 51 | 82 | 0,154918 | 9,87433 | 1,41407 | 0,206384 | 7,41196 | 4,45408 |
| 2 | 0,14156 | -1,2285 | -3,93901 | 1 | 283 | 37 | 24 | 787 | 37 | 38 | 0,161543 | 8,57576 | 1,37883 | 0,209657 | 6,60769 | 3,91194 |
| 3 | 0,22765 | -1,2285 | -3,93901 | 1 | 165 | 38 | 62 | 286 | 54 | 19 | 0,119992 | 14,0766 | 1,37883 | 0,155731 | 10,8461 | 3,91194 |
| 4 | 0,14156 | -1,05401 | -3,93901 | 1 | 192 | 25 | 34 | 124 | 29 | 19 | 0,182236 | 7,60196 | 0,999814 | 0,255499 | 5,42213 | 3,86306 |
| 5 | 0,22765 | -1,05401 | -3,93901 | 1 | 462 | 77 | 47 | 623 | 32 | 27 | 0,135363 | 12,4782 | 0,999814 | 0,189782 | 8,9001 | 3,86306 |
| 6 | 0,14156 | -1,2285 | -3,85821 | 1 | 004 | 64 | 45 | 326 | 45 | 06 | 0,177291 | 7,81398 | 2,00037 | 0,224419 | 6,17307 | 5,13556 |
| 7 | 0,22765 | -1,2285 | -3,85821 | 1 | 065 | 64 | 35 | 824 | 58 | 91 | 0,13169 | 12,8262 | 2,00037 | 0,166695 | 10,1327 | 5,13556 |
| 8 | 0,14156 | -1,05401 | -3,85821 | 1 | 108 | 53 | 62 | 663 | 58 | 67 | 0,200002 | 6,92669 | 1,4505 | 0,273488 | 5,06549 | 5,07141 |
| 9 | 0,22765 | -1,05401 | -3,85821 | 1 | 276 | 68 | 84 | 161 | 33 | 02 | 0,148559 | 11,3697 | 1,4505 | 0,203144 | 8,31469 | 5,07141 |

| Key | Low | High |
|---|---|---|
| D | 0.12 | 0.2 |
| C_g | 100.0 | 100.0 |
| Hen | 0.031 | 0.031 |
| vTip | 1.0 | 1.0 |
| N | 6.5 | 13.0 |
| Q | 1.0 | 2.0 |
| PGV | 1.0 | 1.0 |
| kLam | 1.0 | 1.0 |
| OTRC | 1.0 | 1.0 |
| ReN | 1.0 | 1.0 |

Copy the *Low-Scale* ***x****-design* into the ***MODDE®*** *prediction sheet* to confirm that impeller diameter is now low for both scales ☹.
*A-t-w-k (****A****ll* ***t****hat* ***w****e* ***k****new* and put into the ***u****-formulae)* was not good enough.



# Summary of our Results

+ Scale Down by a *factor if 1,4* (for *impeller diameter*) is possible

– Scale Down for a *factor of 5* is not possible
  (*vTip ~ D, PGV ~* $D^5$ and *OTRC ~* $D^2$ cannot be controlled simultaneously)

+ An orthogonal design for *presumed causal drivers* can be set up

+ Dependencies of other relevant variables are made transparent

– In the procedure described above, D is varied at the low scale:
  Hold *D* constant, this reduces dimension from $2^3$ to $2^2$ for *N* and *Q*

# Thank you!

Gefördert durch:

Bundesministerium
für Wirtschaft
und Energie

aufgrund eines Beschlusses
des Deutschen Bundestages

and special thanks to:

Chhawang Lama
Anthony Orth